Civil war since 1945: Some facts and a theory*

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Abstract

The most common form of civil war in the post-World War II period has been a stalemated guerrilla war confined to a rural periphery of a low-income, post-colonial state. Standard contest models of conflict do not capture important and distinctive features of insurgency, and in particular the fact that guerrilla survival depends on their controlling information about who and where they are. I present a game model in which rebel control of territory depends on how many remain uncaptured by government forces. Capture becomes more likely as the rebel movement expands, due to network connections among the rebels. The model explains how and why insurgencies can remain stalemated at low levels of conflict. It also shows that standard explanations for the strong cross-national association between poverty and civil war risk – for example, that poverty makes joining a rebel band a more attractive option or that risk aversion makes the rich more fearful of conflict – are incoherent or strongly incomplete as typically stated. I argue that more plausible explanations for the empirical regularity pose an indirect link, via the association of high income with (a) natural and social terrains inimical to guerrilla hiding, (b) possibly state military capability to conduct more efficient counterinsurgency, and (c) inability to appropriate as large a share of income through house-to-house visits by guerrillas, due in part to the mobility of human capital.

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1 Introduction

Civil war has been by far the most common and destructive form of violent military conflict in the last 60 years. About 40% of countries with a 1990 population of at least half a million have had at least one internal conflict that killed at least 1,000 people. A large number of countries have had conflicts that killed tens or even hundreds of thousands. Civil war has been the major cause of forced migrations in this period, which have produced many millions of refugees. Overall, the damage to the health, economies and lives of the survivors has been massive (Ghobarah, Huth and Russett 2003).

A growing empirical literature has established a range of interesting facts and regularities about civil war since 1945, including the following.

First, most civil wars in this period have been fought as guerrilla conflicts in which state counterinsurgent forces hunt for small, lightly armed rebel units operating in rural areas, often in rough terrain. Guerrillas tend to “control the night,” while government forces operate by day and in urban strongholds in the war-affected region of the country. There are no clear front lines. Rebel units seek to ambush government units, assassinate or intimidate local government officials and other perceived collaborators, and to raise funds by collecting “revolutionary taxes” of various sorts (for example, from peasants or from businesses). Government forces may threaten or bribe locals for information about rebels, set up paramilitary “village guard” units, force peasants into “strategic hamlets,” or massacre whole villages suspected of aiding or being sympathetic to the rebels. This pattern has been visible in Vietnam, Guatemala, El Salvador, Columbia, Algeria, Philippines (NPA and in Mindanao), southeast Turkey, Peru, northeast India, Burma, Thailand, Kashmir, Nepal, Indonesia (Aceh and East Timor), Mozambique, Sierra Leone, Sudan, and many other places.

Conflicts akin to the U.S. Civil War in which conventional armies fight along well-defined fronts have been quite rare. The Biafran war is the clearest instance. Still, in accord with Mao’s doctrine of “people’s war,” the later stages of some civil wars that began as guerrilla conflicts have tended in this direction (e.g., China 1945-49, Afghanistan, Ethiopia, and Uganda 1981-86). Another occasional pattern is conflict between militias organized on ethnic or regional lines (e.g., Lebanon, Turkey 1977-80, Bosnia, Tajikistan), although these have similarities and overlaps with both guerrilla and conventional civil wars.¹

A second striking regularity is that poor countries have been much more likely to have civil wars than middle-income countries, and middle-income countries much more likely than rich countries. Figure 1 shows how the frequency of civil war outbreaks has varied with per capita income for 161 countries observed between 1945 and 1999 (using data from Fearon and Laitin 2003). Of the richest fifth of the country years in the sample, only about 1.5% had a civil war erupt within 1945.

¹See Kalyvas (2004) for a discussion of types of civil warfare that makes a similar set of distinctions.
the next five years. By sharp contrast, among the poorest fifth of country years, 14.3% had a civil war start in the next five years.

Moreover, per capita income is the single best predictor of a country’s odds of civil war outbreak, empirically dominating other factors that one might have expected to do better, such as level of democracy, degree of ethnic or religious diversity or nature of ethnic demography, or level of income inequality.\footnote{Fearon and Laitin (2003). The finding that level of economic development is a strong (typically the strongest along with total population) correlate of violent civil strife is one of the most robust in the growing large-N literature on the subject, appearing consistently in study after study. Some scholars used related variables such as life expectancy, education levels, or per capita energy consumption, which are closely correlated with per capita income and plausibly tap a common underlying dimension of level of economic development. Cites ..} Indeed, after controlling for income, none of these factors have any added purchase.

There are at least two important theoretical puzzles raised by these facts and regularities.
First, how to explain what is probably the modal civil war in this period – a persistent, small, stalemated guerrilla conflict? What prevents government and rebel groups from cutting deals that would allow them to avoid the enormous costs? Second, what explains the strong association between low per capita income and high civil war risk?

I address these questions with the help of a game model in which a government and a rebel leadership simultaneously choose how many soldiers and rebels to conscript (or to recruit, in some versions). Counterinsurgency by government forces then yields the capture or death of a fraction of the rebel group. Uncaptured rebels proceed to tax peasants or businesses in their region of operation, and to exclude the government from collecting taxes.

The model is related to the “contest models” that Grossman (1991, 2002), Hirshleifer (1995), Skaperdas (1992) and other economists have used to analyze violent conflict, but it differs in two significant respects. First, in standard formulations, contest models of conflict do not actually have any violence in them. No one can get killed or captured. In some models, conflict is assumed to destroy resources, which can be interpreted as an effect of violence. But the implications of conflict for individual fighters are not modelled, making it impossible to analyze individual decisions to participate, or the trade off between risk and potential reward of becoming a rebel. This is a significant liability when the most common “explanations” for why low-income countries are civil-war prone hold that poor young males find the option of rebellion more attractive, or that poor people are generally more aggrieved.

Second, in typical contest models almost all the explanatory action is built into the “contest success function,” which relates the players’ fighting efforts to the final distribution of resources. By separating the extraction of resources from the violent interaction between government and rebel forces, the model here takes a small step towards “unpacking” or opening up the standard contest success function.

One benefit of this approach is that it forces us to consider what is distinctive about the mechanisms of violence in insurgency. I argue that in contrast to ordinary crime and conventional military confrontations, mafias and insurgencies face the problem that adding more fighters raises the risk of detection and thus capture for all existing fighters. In practice, the central problem of counterinsurgency is not to marshal adequate forces to defeat rebel units, but rather to gain good intelligence on who and where the active rebels are. If rebels are linked to each other, then adding more increases risks of infiltration, betrayal, and detection for large parts of the organization. I argue that this information externality of adding more members provides a natural explanation for “diminishing returns” to guerrilla warfare, which is in turn necessary to explain how this kind of deadly conflict can remain small, stable, and stalemated.

On the question of why poor countries are civil-war prone, I argue that common arguments in the empirical literature on civil war do not work. One of the main obstacles is that although richer people may have more to lose from civil strife, the fact that there is more wealth around to
tax or appropriate also means that there is more to be gained by fighting. Conversely, if there is relatively little to fight for, as in an impoverished country, why fight? In contest models, the bigger the “pie,” the greater the equilibrium fighting efforts. This problem tends to undermine one of the most popular arguments proposed in the empirical literature on civil war, namely, that poverty makes for civil war because in poor countries there are more poor, underemployed people who find rebellion attractive as a “job” (e.g., Collier and Hoeffler (2004)).

A possible reply holds that while there may be more to gain from rebellion in a wealthier economy, “diminishing marginal utility,” or risk aversion, means that greater per capita income makes citizens less willing to run risks of capture or death in a civil conflict. I show that by itself this assumption does not solve the puzzle either. One needs to make a stronger assumption about preferences, namely that richer people are relatively more risk averse than poorer people. This assumption flies in the face of empirical work in finance (where richer people typically choose riskier asset bundles), and the spirit of classic works in political science such as Scott (1976). One might “get” a certain amount of risk-seeking behavior by the poorest of the poor if one assumes that starvation is the likely alternative to joining a rebel band, but it seems implausible that this could explain much of the civil war we have seen in the last 60 years.\(^3\)

In the models presented below, the equilibrium level of insurgency and counterinsurgency is strongly influenced by the rebels’ ability to collect taxes and by the government’s ability to capture rebels. I argue that a plausible and internally coherent explanation for the empirical regularity is that in rich countries, large amounts of the income being generated is not easily “taxed” using simple extortion, and in poor countries rebels can more easily evade capture at given force levels, due to terrain, informational advantages of rural village as opposed to urban settings, and possibly government competence in running effective counterinsurgency campaigns.

In the next section I introduce a simple contest model applied to civil war, and show why it produces no “first order” result linking poverty and civil war risk. In Section 3 I discuss differences between the strategic logics of ordinary crime, mafias, insurgencies, and conventional military conflict. The main argument is that as with mafias, adding new rebels to a small insurgency increases detection and denunciation risks for existing rebels, due to their network connections. Section 4 uses this argument in a model that explicitly represents the government’s efforts to capture and kill rebels. Section 5 returns to the question of civil war risk and per capita income, modifying the model to allow for the recruitment of risk-averse rebels rather than conscription. Section 6 considers why more efficient peace deals are not obtainable in equilibrium in the model, and obstacles to efficiency in the case where government and rebels interact over time and thus could in principle support cooperation by implicitly threatening to return to conflict.

\(^3\)Grossman and Mendoza (2003) develop essentially this argument to explain some anthropological observations of an association between resource scarcity and increased conflict in premodern societies where starvation was common.
2 Contest models of civil conflict

In this section I present a simple contest model applied to civil conflict, and discuss its main implications and liabilities.

Consider a game with two strategic players, a government $G$ and a rebel leadership $R$. They interact in a society with a continuum of individuals (normalized to size 1), each of whom has pre-tax income $y > 0$. Total potential tax revenues are thus $ty$, where $t \in [0, 1]$ is a fixed tax rate.\footnote{We could assume that individuals can hide their income at marginal cost $h \in (0, 1)$, in which case government and rebels optimally set tax rates at $t = h$.} Suppose that $R$ and $G$ simultaneously choose to conscript $\alpha \in [0, 1]$ and $\beta \in [0, 1]$ rebels and soldiers, respectively, at marginal costs $c_R$ and $c_G$.\footnote{For completeness assume that if $R$ and $G$ choose $\alpha$ and $\beta$ such that $\alpha + \beta > 1$, then their realized force sizes are $\alpha' = \alpha/(\alpha + \beta)$ and $\beta' = \beta/(\alpha + \beta)$ respectively. This is unimportant however since $\alpha$ and $\beta$ such that $\alpha + \beta > 1$ cannot be best replies given the utility functions below.} Let the “contest success function” be $p(\alpha, m\beta)$, which gives the share of tax revenue controlled by $R$ when the force sizes are $\alpha$ and $\beta$. $m > 0$ is a parameter scaling the effectiveness of counterinsurgency. Rebel and government utility functions are then

$$u_R(\alpha, \beta) = p(\alpha, m\beta)ty(1 - \alpha - \beta) - c_R\alpha \quad \text{and} \quad u_G(\alpha, \beta) = (1 - p(\alpha, m\beta))ty(1 - \alpha - \beta) - c_G\beta.$$

The standard assumption is that, holding the other side’s force level constant, more rebels or more soldiers gets one more territory and tax revenue, though at a diminishing rate ($p_1 > 0$, $p_{11} < 0$, $p_2 < 0$, and $p_{22} > 0$).

Little more than inspection of the utility functions is necessary to understand how varying per capita income will affect equilibrium levels of conflict. Notice that increasing $y$ (without changing anything else) increases the marginal returns to rebellion for $R$, and to counterinsurgency for $G$. That is, if $\alpha(\beta)$ was $R$’s optimal force size given $\beta$ before increasing $y$, then diminishing returns ($p_{11} < 0$) implies that after increasing $y$, $\alpha(\beta)$ will be larger.\footnote{Diminishing returns also ensures that $\alpha(\beta)$ is unique.} The same logic holds for $G$, and the net effect in equilibrium is that higher $y$ implies that a higher percentage of the population will be employed as rebels and soldiers fighting them.

This is the exact opposite of the empirical regularity, but it is a general feature of contest models (including wars of attrition). On first glance it does have a certain logical appeal. Isn’t it natural to think that rational actors would fight harder and more for a bigger prize? Don’t scholars in the civil war literature routinely “explain” the association between oil production (or other natural resources) and civil war by arguing that these increase the value of winning? Why...
wouldn’t the same be true of the size of the economy?

On a second glance, it should be noted that increasing per capita income \( y \) while holding constant \( c_R \) and \( c_G \) is equivalent to saying that the marginal cost of conscripting, feeding, and supplying combatants is lower in rich countries. Not surprisingly, if it is effectively cheaper to man armed forces, equilibrium levels of conflict will be higher. But surely these costs would be higher in richer countries. If we assume that they increase proportionally with income – say the marginal cost of conscription is \( c_i y \) instead of \( c_i \), \( i \in \{G, R\} \) – then varying income clearly has zero effect on equilibrium levels of conflict. In this case, we can divide \( y \) out of the utility functions above without changing \( R \) or \( G \)’s incentives at all.

This result travels across a variety of alternative specifications. For example, suppose that \( G \) and \( R \) have to hire labor rather than conscript it. Since the model as it stands does not involve any risk of death or jail for combatants, both \( G \) and \( R \) can offer a wage of \( w = y \) (or infinitesimally more) to attract fighters, and this clearly leads to equilibrium levels of \( \alpha \) and \( \beta \) being independent of \( y \). Or suppose that incomes vary in the population, with \( y_i \) distributed by a cumulative distribution function \( F \). If the shape of \( F \) does not change as per capita income \( \bar{y} \) changes, we can again factor \( \bar{y} \) out of the expressions for \( u_G \) and \( u_R \), meaning that income level does not affect any marginal trade-offs and thus equilibrium force sizes.

To be more specific, let \( w = (1 - t)F^{-1}(\alpha + \beta) \) be the market-determined wage for rebels and soldiers when \( R \) hires \( \alpha \) rebels and \( G \) hires \( \beta \) soldiers (for simplicity I am assuming that neither rebels nor soldiers pay taxes). At this wage, the \( \alpha + \beta \) poorest fraction of the society prefers to sign up on one side or the other, while the \( 1 - \alpha - \beta \) richer (or more productive) fraction prefers to work in the regular economy. \( u_R(\alpha, \beta) \) then becomes \( p(\alpha, m\beta)t \int_{\alpha+\beta}^{1} F^{-1}(z)dz - w\alpha \). To examine the effect of changing per capita income without changing anything else (such as the shape of the income distribution), we define \( F_0(y/\bar{y}) = F(y; \bar{y}) \) as the “base” distribution. Then the market clearing condition becomes \( \alpha + \beta = F(w(1 - t)/(y(1 - t))); y_i \), and thus \( w = \bar{y}(1 - t)F_0^{-1}(\alpha + \beta) \). Using \( F^{-1}(z) = \bar{y}F_0^{-1}(z) \), substitution into \( u_R \) yields

\[
\begin{align*}
u_R(\alpha, \beta) &= p(\alpha, m\beta)t \int_{\alpha+\beta}^{1} \bar{y}F_0^{-1}(z)dz - \bar{y}(1 - t)F_0^{-1}(\alpha + \beta)\alpha.
\end{align*}
\]

Per capita income can be factored out of the rebel group’s preferences without affecting any trade offs, and so does not affect equilibrium force levels. The same is true for the government.

Changing the shape of the income distribution does affect incentives for rebellion and counterinsurgency in the basic contest model. Holding per capita income constant while increasing inequality lowers marginal recruitment costs for both government and rebels, since there are more relatively poor people around and the total tax base is the same. Thus, greater inequality associates with higher equilibrium force levels.\(^7\) In principle this could help explain the empirical regularity

\(^7\)But not necessarily greater inefficiency, which in this model comes from the fact that rebels and soldiers are not
if richer countries are systematically more equal than poorer countries. The ambiguous empirical support for the Kuznet’s curve suggests that there is some tendency in this direction, but it is not very strong. Also, as noted above, more unequal countries have not been more prone to civil war in the last 60 years, at least using standard cross-national inequality measures.

On the other hand, this analysis suggests that the “bigger prize” argument for why oil producers appear to have been more civil war-prone does not work unless recast as an argument about oil making for big inequalities. If oil revenues are monopolized by a small group that controls the state, then $\bar{y}$ is large relative to the marginal cost of recruiting rebels and soldiers, which favors larger rebellions, according to the model.\(^8\)

To sum up, the contest model approach points to two main effects of per capita income on the propensity for civil conflict. On the one hand, more income means more revenue for rebels to appropriate and government forces to defend. But on the other hand, the marginal costs of staffing a rebel or government force will be greater in a richer country. These effects work in opposite directions, tending to give the result of no net impact. This is a simple point, but I have not seen it stated or explored in the literature applying contest models to conflict.\(^9\)

The basic contest model does suggest two second-order ways that low income might plausibly cause higher equilibrium levels of civil conflict. To see the first, note that the argument above implies that if you give the same amount of counterinsurgency funding to a poor and rich state, the poor state should get a much bigger “bang for the buck” because it would be able to hire (or support) much more labor. Much anecdotal and case-based evidence suggests, by contrast, that richer countries are more efficient at using counterinsurgency funds, either because of higher levels of human capital, training and organizational coherence, or because the strategic and tactical problems are more easily solved in an economically more developed setting (see below), or both.

Second, it could be that individuals and businesses are less able to hide their income from insurgents in poor than in rich countries. The standard “appropriative technology” of insurgency consists of visits to households or businesses to collect revolutionary taxes, often in kind. This producing what they could. Greater inequality at a given per capita level means that there are more rebels and soldiers, but they would have been less productive in the regular economy anyway. Adding direct damage to the economy from conflict would change this, however, and in practice these effects are surely far larger than that of labor displacement.

\(^8\)Fearon and Laitin (2003) and Fearon (2005) argue that oil exports favor civil war by increasing the “prize” value of capturing the state or region, and because, conditional on income level, oil producers tend to have less developed state administrative apparatuses and capabilities. Humphreys (2005) considers a broader array of possible mechanisms. Olsson and Fors (2004) consider a contest model of civil conflict in which the ruler controls natural resource rents; greater resource rents make for more conflict in their model, by the same logic as that described here.

\(^9\)Grossman and Mendoza (2003, 747-48) may be alluding to the issue when they write that “Surprisingly, the commonsensical hypothesis that resource scarcity causes a large allocation of time and effort to appropriative competition is not easy to formalize,” and that “there is no reason to presume that in general the relative return to appropriative competition either increases or decreases with the size of the resource endowment.”
may yield a higher share of total product in a society of small-holding peasants than in a world of mega-corporations and mobile, high-income-from-high-human-capital workers. If so, the effective tax rate insurgents can impose would be higher in poor countries. It is easily shown in the contest model that higher tax rates lead to higher equilibrium levels of conflict.\textsuperscript{10}

The standard contest model is a highly “reduced form” approach to analyzing conflict. It hides the specifics of the interactions between combatants inside the “black box” of the contest success function. With minor changes, the model could be redescribed as a model of interstate conflict, conflict between animals for territory, between firms for market share, between candidates for votes, or between lobbyists for policy. In all such cases, results follow from embedded assumptions about the shape of the contest success function, and in particular the assumption of diminishing returns to effort and an assumption about the specifics of the cross-partial derivative of $p(\cdot, \cdot)$.\textsuperscript{11}

The range of application of the contest model is in one respect a virtue, in that it highlights the strategic similarities of a broad range of political and economic situations. But it can also be a vice if the contest success function obscures important distinctions between types of conflict and violence. In the case at hand, we should not accept without argument that the returns to rebellion are diminishing (which makes possible a stable, interior equilibrium with a low level of insurgency in the contest model above). Many discussions of rebellion assume to the contrary that there are increasing returns, for example, a tipping point beyond which the government will fall. At a minimum we need a developed argument for why the returns to insurgency would be diminishing, and in the contest model approach this will at best be given “off stage.”

Below I offer a model that opens up the contest success function, distinguishing between the violent interaction between government and rebel fighters, and the interaction between surviving rebels and peasants over revenue. To justify some important assumptions that go into this model, however, I need first to discuss how the strategic logic of insurgency differs from several other kinds of conflict that have been modelled using contest success functions.

\textsuperscript{10}Along similar lines, external funding from neighboring states and superpowers has been an important source for insurgencies since 1945, acting in a way similar to raising $t$ or lowering $c_R$ in the contest model. Another possible explanation for the concentration of civil wars in poorer countries since 1945 is that civil war has been a form of interstate war by proxy, and that the relative absence of civil war among richer countries is a by product of the factors that have favored interstate peace among richer countries in this period.

\textsuperscript{11}The usual assumption is that $p_{12} > 0$ for $\alpha > m\beta$ and $p_{12} < 0$ for $\alpha < m\beta$. This gives rise to best reply functions that increase and then decrease, and which intersect at an $(\alpha, \beta)$ such that $\alpha = m\beta$ when $p(\cdot, \cdot)$ is symmetric.
3 Crime, mafias, insurgencies, and conventional warfare

In ordinary property crime, maintaining anonymity is the criminal’s core strategy for avoiding arrest. Criminals seek to burgle unseen, to hold up banks wearing masks, or to mug quickly and then disappear into a large, anonymous urban population. They operate as individuals or in very small groups. The central motif of almost all crime drama is the finding and proving of “who done it?”

The strategy of anonymity means that repeat business is not an option for ordinary criminals. They cannot repeatedly mug the same people, or return repeatedly to the same store or bank, without being identified and arrested. Mafias are an attempt to solve this problem. Mafia members must make themselves known to the individuals and business owners from whom they extort regular payments. Their strategy for avoiding arrest is then to threaten violent reprisal if their victims denounce them to the police, and in particular if they testify in court. To make such threats credible, a mafia requires, in the first place, an organization. If one member is arrested due to testimony by a victim, there must be other members willing and able to punish the victim.

And given that there must be multiple members, there is a huge premium on loyalty within the organization. Since members know about and participate in the organization’s violence and violent threats, each member poses a denunciation risk to everyone other member. Thus the sacred oaths, long initiation periods, ethnic and family ties, draconian punishments for suspected informers, and witness protection programs as an anti-racketeering strategy. Whereas increasing the total number of ordinary criminals may increase the expected returns to any given criminal (since the police efforts are more diffused), adding mafiosi may decrease the expected returns to existing members because the network connections make for negative externalities regarding infiltration and betrayal.

Because mafiosi necessarily make themselves known to their targets, the problem for police is not primarily in identifying who the mafiosi are, but rather in acquiring solid evidence of criminal activity. They may face problems in locating a mafioso for arrest, but at least in urban environments this is generally not so difficult. One implication is that if the state does not care that much about evidence or due process and its agents are relatively un briable, mafias cannot survive. Such a state will just kill or throw suspected racketeers in jail. Mafias did not flourish in Eastern Europe and much of the former Soviet Union until after the demise of communist regimes. Mafias require either a political environment with the rule of law and due process, or a state whose agents can be bribed to look the other way (or a combination of the two).

Insurgencies differ from mafias in espousing political goals. They seek either to become the formally recognized government in a region, to replace the current government in the country or to force a change in its policies.¹² Their situation also differs from mafias in that even rule-

¹²There are cases, such as the RUF in Sierra Leone, where it is not clear whether the insurgent leaders are sincere
of-law countries tend to have few scruples about attacking and detaining rebels without full due process. Combined with the fact that insurgencies start out and often remain small and militarily weak relative to government forces, this means that they have to be able to hide from government troops and intelligence. Rural settings with close access to mountains or jungles are thus strongly favored.

But rough terrain is rarely sufficient to allow the survival of a guerrilla band, since the guerrillas must interact with people at least some of the time. They typically draw food and funds in the form of revolutionary taxes on households and businesses, engaging in “repeat business” with the same villages and people. They require locals’ information about government troop movements, and their own movements and activities are partly observed by local noncombatants. This means that a successful insurgency must address the same core problem that mafias face – the risk of denunciation to authorities. And like mafias, insurgents almost invariably threaten and carry out violent punishments against those who denounce (or are said to have denounced), in order to deter others (Kalyvas 1999; Kalyvas 2003). They may also provide more positive inducements to lessen the risk of denunciation, such as ideological training programs, control and discipline of individual rebels who injure civilians, and defense against marauding government troops. But the government’s willingness to use force against civilians to acquire information about the rebels tends to quickly draw any rebel group into a competition of threats and violence with respect to noncombatant locals.

As with mafias and in contrast to ordinary crime, adding new members to a small insurgent band raises denunciation and detection risks for the whole organization. More rebel units are more likely to be seen by locals or to try to “tax” individuals who are willing to run the risk of reporting to the government. Because rebels are linked to each other and because in their early stages guerrilla movements depend on being able to hide, the capture of one rebel can favor the capture of more. In addition, monitoring and screening become more difficult as the size of the rebel organization expands. It is reasonable to suppose that the ideological commitment of additional rebels is diminishing (the more intensely committed types are already in the organization), which means that expansion increases the risks of informers, defections, and bad types who raise detection risks about their political objections, but they nonetheless espouse them.

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13External funding from neighboring states, the U.S. and U.S.S.R. during the Cold War, or from ethnic diasporas are also quite significant for many insurgencies. See Weinstein (2005) and Hovil and Werker (2005) for studies of the implications of external funding for insurgent strategy and tactics, and Byman et al. (2001) for a study of external funding of insurgent groups after the Cold War.


15More evidence in favor of this characterization is provided by the cell structures that rebel organizations have consistently tried to employ to lessen the network externalities of detection and infiltration.
by overly abusing noncombatants.\textsuperscript{16}

In sum, network connections and the need to hide imply that individual rebels may be made less safe rather than more safe when the rebel organization increases in size.\textsuperscript{17} By contrast, ordinary criminals and soldiers in a conventional army are made more safe when more criminals or soldiers are added (holding police or the other military constant). In the case of criminals, police efforts are more diffused. In the case of a conventional military, a bigger army is more likely to win, and at a lower cost in lives.

The model of insurgency in the next section uses the assumption that adding rebels increases the share of the total rebel group that is captured or killed for a given government force size. As noted above, in both Mao’s theory and a number of internal conflicts in the last 50 years, small guerrilla movements have grown so large that they were able to reconfigure themselves as conventional military forces able to fight set-piece battles against state armies. A more sophisticated model might incorporate this possibility.

4 A model of insurgency

Again we consider a game with two strategic players, a government $G$ and a rebel leadership $R$. They compete over control of tax revenues from a continuum of individuals (normalized to size 1), each of whom has pre-tax income $y > 0$. However in this case, to make the model more tractable and transparent, I will assume that the continuum of individuals represents people living in a particular region of the country, and that $R$ is incapable of or uninterested in expanding control beyond this region. $R$ draws its conscripts (or later, recruits) only from the region, whereas $G$ draws conscripts or recruits from outside the region.\textsuperscript{18} Let $\alpha \in [0, 1]$ be the size of the rebel force, and $\beta \geq 0$ be the size of government forces. Total potential tax revenues from the region are $ty$.

\textsuperscript{16}Rebel organizations may devote considerable resources to ideological training of recruits (e.g., NPA in the Philippines, Museveni in Uganda, ...), or use lower tech methods such as requiring recruits to kill soldiers or even members of their own family (to make it very difficult for them to return to their former life).

\textsuperscript{17}Here is a simple illustration of how network connections will tend to make the probability of capture increasing in the size of the organization. Consider a country or region with $n$ people, $r$ of whom are active rebels. The government interrogates $s$ people at random. Let the probability of getting “good information” be $q \in (0, 1)$ if it interrogates an actual rebel, and zero otherwise. If the government gets good information, it captures a share $\gamma$ of the $r$ rebels in the area. Then a rebel’s probability of capture is $\gamma \sum_{j=0}^{r} B(j; s, r/n)(1 - (1 - q)^j)$, where $B(j; s, r/n)$ is the probability that there are $j$ rebels in a random sample of $s$ people. This is increasing in $r$, the number of rebels.

\textsuperscript{18}For example, we could think of the model as representing a conflict between a secessionist rebel group in an ethnically homogenous region. Alternatively, we could imagine that the continuum represents the whole country, but that the government’s soldiers are mercenaries from another economy. It should be stressed that this assumption is completely immaterial to any of the results that follow; they will also obtain for the more complicated model in which both government and rebels recruit from the same pool.
where $t \in [0, 1]$ is a fixed tax rate.

The sequence of actions and events in the game are as follows.

1. $R$ and $G$ simultaneously choose to conscript $\alpha$ and $\beta$ rebels and soldiers, at marginal costs $c_{Ry}$ and $c_{Gy}$, respectively.

2. A fraction of the rebels, $p(\alpha, m\beta)$, are captured or killed by government forces.

3. The remaining rebel force, now of size $\alpha(1 - p(\alpha, m\beta))$, collects revolutionary taxes from the regional population who are not in the rebel group. Assume that one rebel collects from $\delta > 1$ peasants, so that $R$’s total revenues are the smaller of $ty(1 - \alpha)$ and $ty\delta\alpha(1 - p(\alpha, m\beta))$.

4. The government collects tax revenues from peasants who are not “controlled” (here, taxed), by rebels. Thus $G$’s total revenues from the region are the larger of zero and $ty(1 - \alpha - \delta\alpha(1 - p(\alpha, m\beta))$.

Two differences from the standard contest model formulation should be stressed. First, notice that the model separates the interaction between insurgents and counterinsurgents from the interaction between insurgents and locals that produces revenues. Accordingly, $p(\alpha, m\beta)$ is no longer a contest success function, but might instead be called a “capture” or “capture and kill” function, since it relates the size of rebel and government forces to the share of rebels captured or killed. We assume that $p(\alpha, m\beta)$ is increasing in both its arguments. This is uncontroversial for $\beta$, since this just means that more government forces capture or kill a larger share of a given rebel force. The assumption that the share captured or killed increases as well with the size of the rebel force follows on the argument given above, that denunciation and detection are critical for rebel losses, and are harder to prevent as force size grows.

In the appendix, I consider the model with an arbitrary function $p(\alpha, m\beta)$ that is increasing in both arguments. For clarity and convenience I will work here with the specific functional form

$$p(\alpha, m\beta) = \frac{m\alpha\beta}{m\alpha\beta + 1}.$$

A second feature worth noting is the assumption that the rebels’ tax collection technology has constant returns to scale up to the point at which the whole region is controlled by the rebels. One can imagine arguments for why there would be increasing returns (e.g., government is a natural monopoly), or decreasing returns (e.g., eventually there must crowding, as occurs in this model), or perhaps increasing then decreasing returns. I don’t see a decisive consideration one way or the other. Clearly, though, one way to “get” decreasing returns to rebellion would be to simply assume that the rebel’s collection technology has quickly decreasing returns to scale, whereas the government’s does not.
Rebel and government utility functions are thus

\[
\begin{align*}
    u_R(\alpha, \beta) &= ty \min\{1 - \alpha, \delta\alpha(1 - p(\alpha, m\beta))\} - c_R y \alpha \quad (1) \\
    &= ty \min\left\{1 - \alpha, \frac{\delta\alpha}{m\alpha + 1}\right\} - c_R y \alpha \quad (2)
\end{align*}
\]

and

\[
\begin{align*}
    u_G(\alpha, \beta) &= ty \max\{0, 1 - \alpha - \delta\alpha(1 - p(\alpha, m\beta))\} - c_G y \beta \quad (3) \\
    &= ty \max\left\{0, 1 - \alpha - \frac{\delta\alpha}{m\alpha + 1}\right\} - c_G y \beta \quad (4)
\end{align*}
\]

**Analysis.** Figures 2 and 3 graph two possible configurations for the rebels’ and government’s best reply functions. \(\alpha(\beta)\) is the rebel leadership’s optimal force size given that there are \(\beta\) counterinsurgency personnel, and \(\beta(\alpha)\) is the government’s optimal force size given that there are \(\alpha\) rebels. Figure 2 represents a case with a unique pure strategy Nash equilibrium (at the intersection of the two curves), and Figure 3 a case with no pure strategy equilibrium.

A good way to grasp the logic of model is to work through the logic of the best reply functions. Beginning with the rebels, if there is no counterinsurgent effort \((\beta = 0)\), then \(R\) can tax the whole regional population by setting \(\alpha\) such that \(\alpha + \delta\alpha = 1\), or \(\hat{\alpha} = 1/(1 + \delta)\). \(R\) does not want to conscript more than \(\hat{\alpha}\), since this would increase costs without increasing revenues. And it is optimal to choose \(\hat{\alpha}\) provided that the marginal conscription cost \(c_R y\) does not exceed the marginal revenue produced by an additional rebel, \(ty\delta\). I will assume in what follows that \(c_R < t\delta\), which
says that forming a rebel band/mafia/local government would certainly be worthwhile if there were no central government response.

If $β > 0$ there are two possibilities. Either $u_R$ is maximized by an $α$ such that $\frac{δα}{mβ+1} < 1−α$, or not.\textsuperscript{19} If this condition holds, then $R$’s marginal returns equal the marginal cost of conscription at a force size such that $R$ does not control the whole region. In this case, the optimal $α(β)$ is given by the solution to the first order condition

$$\frac{ty}{(mαβ + 1)^2} - c_Ry = 0,$$

or

$$α(β) = \frac{1}{mβ} \left[ \frac{tδ}{c_R} - 1 \right]. \quad (5)$$

When the condition does not hold, $R$ wants to choose the force size such that, after conflict, it will have just enough rebels to control the whole region. In this case $α(β)$ is the positive solution to the quadratic resulting from $1−α = \frac{δα}{mαβ+1}$, or

$$α(β) = \frac{1}{2mβ} \left[ \sqrt{(1+δ-mβ)^2 + 4mβ} - (1 + δ - mβ) \right]. \quad (6)$$

The rebel group’s best reply function “flips” from (6) to (5) where the two curves meet, as shown in Figures 2 and 3.

In words, more government soldiers reduce the marginal return to adding rebels by increasing detection risks. When there are “enough” soldiers this means that increasing the counterinsurgency effort leads the rebel group to deliberately contract. Contraction allows them to hide more easily, increasing their survival rate enough to compensate for the smaller base numbers. By contrast, when the rebel group completely dominates the region, increasing government forces initially leads the rebel group to conscript more rebels. In this case the marginal return of additional rebels is still positive; the rebel group had not expanded further because there was no more available tax or support base for it. Increasing counterinsurgency in this case gives $R$ an incentive to expand simply to replace lost rebels so it can collect from the whole region. Eventually the army presence is large enough that $R$ preserves more rebels by contracting.\textsuperscript{20}

\textsuperscript{19}Diminishing returns implies that there is a unique maximum.

\textsuperscript{20}In contest models, the players also have best reply functions that increase and then decrease, but the reason is different. Applied to this setting, the functional form of standard contest success functions would simply assume that greater counterinsurgency increases the marginal return to insurgency when there are fewer soldiers than rebels, but decreases it when there are more soldiers than rebels (e.g., Skaperdas 1992, 724-5). In the specific case analyzed in the text, the marginal return to adding rebels, $\frac{∂}{∂α}tyδα(1−p(α,mβ))$, is always decreasing in $β$, which implies that $α(β)$ is negatively sloped until the constraint is reached. See the appendix for more.
Now consider the government’s optimal level of counterinsurgency given $\alpha$. Ignoring the maximum condition for the moment, the government’s payoff (4) is increasing or decreasing in $\beta$ as

$$t\delta \frac{\alpha^2 m}{(m\alpha\beta + 1)^2} \leq c_G. \quad (7)$$

The left-hand side, the government’s marginal return from counterinsurgency, is decreasing in $\beta$.

So if the left-hand side is less than the right when $\beta = 0$, $G$’s best reply must be $\beta = 0$. This is the case whenever $\alpha$ is less than $\bar{\alpha}$ defined by $\bar{\alpha} = \sqrt{c_G/\delta mt}$. So $\beta(\alpha) = 0$ for $\alpha < \bar{\alpha}$.

In words, if the rebel group is small enough, the government effort needed to get a decent capture probability is too great to make counterinsurgency worthwhile. (Of course, this won’t occur in equilibrium because the rebels want to expand if there is no government resistance.)

For $\alpha > \bar{\alpha}$, the government’s best response may be to set $\beta$ to satisfy (7) with equality, or it may be to set $\beta = 0$. Which is best depends on whether the government can get positive net tax revenues from the region by spending on counterinsurgency. If the government is going to try this, the optimal force size equates marginal revenues and marginal costs in (7), yielding, for $\alpha > \bar{\alpha}$,

$$\hat{\beta}(\alpha) = \sqrt{\frac{\delta t}{mc_G}} - \frac{1}{m\alpha}. \quad (\text{with } \hat{\beta}(\alpha) \text{ increasing in } \alpha)$$

which comes from the assumption (embedded in this specific capture function) that the marginal effect of adding soldiers on the number captured increases as the rebel force size increases.

The question now is whether choosing $\hat{\beta}(\alpha)$ yields a net profit for the government in the region. Formally, is $u_G(\alpha, \hat{\beta}(\alpha)) > 0$ for $\alpha > \bar{\alpha}$? Substituting $\hat{\beta}(\alpha)$ into $u_G$, we have

$$u_G(\alpha, \hat{\beta}(\alpha)) = ty \max\left\{0, 1 - \alpha - \sqrt{\frac{\delta c_G}{mt}} \right\} - c_G\left[\sqrt{\frac{\delta t}{mc_G}} - \frac{1}{m\alpha}\right].$$

From inspection it is evident that government profits from attempting counterinsurgency are decreasing in rebel force size, $\alpha$. Substituting $\alpha = \bar{\alpha}$, algebra yields a condition that must be satisfied if $u_G(\alpha, \hat{\beta}(\alpha)) > 0$:

$$\frac{(1 + \delta)^2}{\delta} < \frac{mt}{c_G}. \quad (8)$$

If this inequality fails, $\beta(\alpha) = 0$ for all $\alpha \geq 0$. This is the case where counterinsurgency is so inefficient or expensive relative to the potential tax revenue that the government neither wants to chase after a tiny rebel force, nor wants to fight back a larger one. It is easy to show that condition (8) fails if and only if $\hat{\alpha} < \alpha$, which is to say that even if the rebels control the whole region the government finds counterinsurgency not worth the effort.
On the other hand, if (8) holds, then there is a range of rebel force sizes, \((\alpha, \bar{\alpha})\), such that the government wants to deploy a counterinsurgent force. The upper bound, \(\bar{\alpha}\), is the \(\alpha\) that satisfies \(u_G(\alpha, \hat{\beta}(\alpha)) = 0\), and is the positive solution to another quadratic (see Proposition 1 below). For \(\alpha > \bar{\alpha}\), counterinsurgency is not worth the effort for the government; it prefers to concede the region, setting \(\beta = 0\). Thus we have the best reply function \(\beta(\alpha)\) indicated in Figures 2 and 3.

A striking feature of \(\beta(\alpha)\) is that it is discontinuous. The government wants to fight harder as rebel force size increases up to a certain point, and then just quits.\(^{21}\) Where does this come from?

Suppose the rebel force is large enough that, unmolested, it has more than enough fighters to control the entire regional population: \(\alpha > \hat{\alpha} = 1/(1 + \delta)\). In this case, the government does not even begin to see any positive returns from counterinsurgency until it has deployed enough troops to reduce the rebel force size below \(\hat{\alpha}\), at which point it can begin to collect some taxes itself. Thus in this case \(G\)'s payoff for no counterinsurgency is zero, and then falls initially for \(\beta > 0\). It may then begin to recover if the marginal returns of reclaiming control are high enough, and it may even recover enough that there is a positive level of counterinsurgency that yields a net profit. At \(\alpha = \bar{\alpha}\), however, there are enough rebels that the optimal counterinsurgency effort \(G\) would yield a net “profit” of exactly zero.

Equilibrium and comparative statics. Proved in the Appendix, Proposition 1 characterizes the Nash equilibrium in the game for different parameter conditions.

Proposition 1: Let \(A = \sqrt{\delta c_G / tm - 1/2}\), and define \(\bar{\alpha} = \sqrt{A^2 + c_G / tm - A}\). The game described above has a unique Nash equilibrium in which \(R\) and \(G\) choose, respectively, \(\alpha^*\) and \(\beta^*\), defined as follows:

1. If \((1 + \delta)^2 \geq \frac{mt}{c_G}\), then \(\alpha^* = \frac{1}{1 + \delta}\) and \(\beta^* = 0\).

2. If \((1 + \delta)^2 < \frac{mt}{c_G}\) and \(\bar{\alpha} \geq \sqrt{\frac{c_G}{mc_R}}\), then
   \[
   \alpha^* = \sqrt{\frac{c_G}{mc_R}} \quad \text{and} \quad \beta^* = \frac{\sqrt{\delta t} - \sqrt{c_R}}{\sqrt{mc_G}}.
   \]

3. If \((1 + \delta)^2 < \frac{mt}{c_G}\) and \(\bar{\alpha} < \sqrt{\frac{c_G}{mc_R}}\), then \(\alpha^* = \bar{\alpha}\) and \(G\) mixes, putting probability \(b^*\) on \(\beta = \hat{\beta}\), and probability \(1 - b^*\) on \(\beta = 0\), where
   \[
   b^* = \frac{t + c_R}{t + \frac{c_G}{m\bar{\alpha}}} \quad \text{and} \quad \hat{\beta} = \sqrt{\frac{\delta t}{mc_G} - \frac{1}{m\bar{\alpha}}}.
   \]

\(^{21}\)At \(\alpha = \bar{\alpha}\), \(G\) is indifferent between choosing \(\beta = \hat{\beta}(\bar{\alpha})\) and \(\beta = 0\), but strictly prefers either to any force level in between these two.
In the first case in the proposition, the government concedes the region to the rebel group, which conscripts just enough labor to tax the population. This is more likely to occur when the rebels are relatively efficient at controlling the population ($\delta$ is large); government counterinsurgency is relatively inefficient at getting a good capture probability ($m$ small); conscription is costly for the government ($c_G$ large); and when the region is not that valuable in terms of the amount of resources that can be extracted ($t$ small).

In the second case, there is a unique pure strategy equilibrium in which the government undertakes an active counterinsurgency and in which the rebels do not conscript enough manpower to control the whole region. This is the empirical case that is the focus of the paper: A small guerrilla conflict that is stalemated and “in equilibrium” in the sense that neither government nor rebels want to increase or decrease their effort given the other side’s effort. For the rebels, adding fighters would increase denunciation and detection rates enough that territory would be lost, or added territory would not compensate the costs. For the government, increasing counterinsurgency effort would yield more rebel captures and more territory under government control. But the rebel group is small and well-hidden enough that the gains would not pay for the costs.

The comparative statics of this equilibrium are most easily read from the best reply functions in Figure 2. Factors that increase the government’s marginal benefits or reduce its marginal costs for counterinsurgency shift the positive portion of $\beta(\alpha)$ upwards.\footnote{They also expand the interval $(\alpha, \bar{\alpha})$.} Likewise, factors that increase the rebel leadership’s marginal returns or reduce its marginal costs for insurgency shift the upper portion of $\alpha(\beta)$ to the right.

Thus, changing a parameter that increases returns for the government but does not affect the rebels’ marginal incentives leads to an increase in the equilibrium size of government forces and a decrease in the size of the rebel force. This is the effect of lowering the government’s costs for conscripting and provisioning soldiers ($c_G$).

By contrast, changing a parameter that increases returns to rebellion without affecting the government’s marginal incentives produces an increase in both rebel and government force sizes. This is the effect of lowering the rebel group’s costs for conscription and provisioning, $c_R$. The government increases its effort in response to changes that favor that rebellion because an increase in the number of rebel units increases denunciation and detection and thus capture opportunities, implying an increase in marginal returns to counterinsurgency.

There are also parameters that affect incentives for both government and rebels, shifting both best reply functions at once. For example, increasing the share of income that can be extracted from the population ($t$) increases the marginal returns to both insurgency and counterinsurgency. This leads to an equilibrium increase in government force size, but no change at all in the size of the rebel force. While rebellion is more attractive given higher $t$, the increased government response
offsets this effect.

Likewise, notice that increasing the rebel leadership’s efficiency or ability to use rebels to extract resources from the population (\(\delta\)) makes both rebellion and counterinsurgency more appealing. For the government, the reason is that if fewer rebels can control the same population, then taking out the same number of rebels has a bigger effect on what the government controls. This factor works the same way as tax yield. Greater rebel efficiency increases the equilibrium level of counterinsurgency, but leaves unchanged the equilibrium size of the rebel force.

Finally, consider increasing the efficiency of government counterinsurgency, \(m\), which might result from either features of the natural or social terrain that aid government forces, or organizational and doctrinal capabilities of the government. Increasing \(m\) shifts the rebels’ best reply function to the left, which by itself would decrease equilibrium insurgency and counterinsurgency. Increasing \(m\) affects the government’s best reply function in a more complex manner, shifting it up for \(\alpha < 2\alpha\), and down for \(\alpha > 2\alpha\). The net effect, however, is to reduce both \(\alpha^*\) and \(\beta^*\).

The third case in Proposition 1 is a mixed strategy equilibrium that obtains, in a sense, “in between” the other two cases. Here the rebels are not so advantaged that the government is willing to let them control the region using the minimum necessary staff (case 1). In case 3, the government prefers to attack if \(\alpha = \hat{\alpha}\). But unlike case 2, the rebels are sufficiently advantaged that they are willing to escalate to levels such that the government would find counterinsurgency to take back a share of the region a losing proposition. Mixed strategies are then necessary for equilibrium, with the implied “cycle” being (a) rebels choose just enough to control region, (b) government wants to attack at this level, (c) rebels want to increase force size above the level needed to control region, (d) government not willing to attack given that size rebel force, and (e) back to (a). In the equilibrium, the rebels choose \(\alpha = \bar{\alpha}\), which leaves the government indifferent between trying counterinsurgency and letting the region go, and thus willing to attack with a probability that makes \(\alpha = \bar{\alpha}\) a best reply for \(R\).

5 Recruitment, risk attitudes, and income

The model of the last section assumed that government and rebels conscript and provision their fighters at a fixed marginal cost. Conscription is certainly common for government forces involved in a civil conflict, and there are many examples of rebel forces effectively conscripting manpower as well. Child soldiers, who are frequently abducted or otherwise forced to join, are remarkably common in rebel groups.\(^{23}\)

Nonetheless, conscription is more problematic for rebels than for the government, since,

---

\(^{23}\)Governments also frequently conscript child soldiers. data ...
especially in the early stages, a rebel leadership does not control an administrative apparatus that can openly monitor a territory for deserters. Further, because hiding is so critical for the survival of guerrilla bands, the risks involved in conscripting fighters against their will are large and often unmanageable. Disgruntled conscripts would be inclined to defect to the government side, bringing information with them.\textsuperscript{24}

So, especially in their small or early stages, guerrilla groups may depend on volunteers far more than conscripts. And volunteers need to be compensated, whether by the prospect of victory and subsequent reward, by the satisfaction of fighting on the side of justice, or by the living that being a rebel provides. Regardless of the first two, joining an insurgency has to provide some material living to rebels since it is a long-term endeavor. Certainly potential rebels will take into account the comparison between the living provided by insurgency and the living provided by the regular economy, even if they also factor in considerations of justice and prospects for changing the government.

The game described above is easily modified so that individuals in the region choose whether to join the rebel leadership in their fight. To keep with the spirit of a “pure” political economy model, I will consider potential rebels who care only about maximizing expected utility from income. They do not have idealistic motivations.\textsuperscript{25}

Suppose that the rebel leadership $R$ offers a wage $w$, which can be enjoyed only if the rebel is not captured or killed. Thus, individuals join the rebellion if their expected utility for the lottery on $w$ and being killed or captured is at least as high as their utility for the income $y$ they can earn in the regular economy. Let $u(x)$ be a concave, strictly increasing utility function for income, with $u(0) = 0$ set as the value for being killed or captured. For simplicity and broadly consistent with the facts, I will continue to assume that government soldiers are conscripted and provisioned at marginal cost $c_{G}y$.

Expected utility for joining the rebels is thus $u(w)(1 - p(\alpha, m\beta))$, while the utility for not joining is $u(y)$. Given government force size $\beta$, if $R$ wants a force size of $\alpha$, it needs to offer a wage of

$$w = u^{-1}\left(\frac{u(y)}{1 - p(\alpha, m\beta)}\right).$$

The wage is increasing in $\alpha$ because the risk of capture increases with $\alpha$ for the reasons discussed above.

\textsuperscript{24}We would expect to observe that when rebel groups do conscript fighters, they have a credible threat against the fighters’ families should their sons or daughters defect. LTTE examples ...

\textsuperscript{25}I think these could easily be incorporated at the expense of more notation, and I doubt the implications would be interesting or surprising (that is, more idealism will imply more rebels and more counterinsurgency).
The rebel group’s utility function then becomes

\[ u_R(\alpha, \beta) = u(ty \min\{1 - \alpha, \delta \alpha(1 - p(\alpha, m\beta))\}) - w\alpha. \] (9)

In terms of equilibrium logic, switching from conscription to recruitment does not change anything important. The only difference is that now the rebel group’s marginal costs for adding rebels are increasing rather than constant. The effect will be similar to shifting \( \alpha(\beta) \) left in Figure 2, implying fewer rebels and a smaller counterinsurgency in equilibrium.\(^{26}\)

However, incorporating recruitment does allow us to analyze an initially plausible counter to the observation made above that while poor people have less to lose by rebelling, they also have less to gain (so why would there be any net effect?) Perhaps the marginal utility of additional income is smaller for wealthier people, making them less willing to risk the very bad outcomes of capture or death as a rebel. By contrast, for poorer people the possible gains from becoming a rebel are much more meaningful.

Another version of this argument is the claim that poverty itself makes for grievance and in consequence a greater willingness to take up arms.\(^{27}\) This is equivalent, I believe, to the proposition that poorer people are more risk acceptant. The idea is that they are willing to run higher risks of capture and death for the same proportional increase in income.

Consider the specific risk averse utility function \( u(x) = x^\rho, \rho \in (0, 1) \). This implies

\[ w = \left( \frac{y^\rho}{1 - p(\alpha, m\beta)} \right)^{1/\rho} = \frac{y}{(1 - p(\alpha, m\beta))^{1/\rho}}. \]

Yet again, \( R \)'s marginal costs for adding rebels are linear in \( y \), and thus \( R \)'s net revenues are as well. \( R \)'s marginal trade-offs concerning \( \alpha \) are not affected by increasing \( y \), and so the equilibrium force size for both rebels and government will again be independent of per capita income, despite our allowing for any degree of risk aversion. The only evident change from introducing risk aversion is that the rebel organization has to pay recruits more to compensate them for the risk of being captured. This will shift \( \alpha(\beta) \) to the left in Figure 2, lowering both \( \alpha^* \) and \( \beta^* \).

The reason that the levels of rebellion and counterinsurgency are independent of income here is that the utility function \( u(x) = x^\rho \) exhibits constant relative risk aversion. If a person’s preferences satisfy this property, then her level of income \( y \) does not affect how she chooses between

\(^{26}\)Analytic solutions are harder to obtain now; even with \( u(x) = x \), \( R \)'s first-order condition requires solving a cubic equation. Note also that if we allow income to vary in the population, this creates another source of increasing costs for rebel recruitment, since they will draw first from the poor and have to offer increasing amounts to persuade additional recruits to join up.

\(^{27}\)Ted Gurr (cite?), and Tanja Ellingsen in conversation.
getting $y$ for sure and a gamble that gives $\tau_1 y$ with some probability, and $\tau_2 y$ with the complementary probability, where $\tau_1 > 1 > \tau_2 \geq 0$.

If we think that increasing relative risk aversion is empirically plausible – that higher income makes one less willing to take a gamble such as, say, a $p$ chance of a 10% increase in income and a $1 - p$ chance of a 40% decline – then we have a possible explanation for the empirical regularity that poor countries are much more civil war prone. The idea is that poor people are more willing to risk steep declines in their income or welfare for the prospect of a proportional gain. Put this way, however, the proposition sounds counterintuitive. The typical assumption in both political science and economics is that poorer people are relatively more risk averse, not more risk acceptant. This assumption could be wrong, or there could be specific features of the “lottery” of joining a rebel band that lead people to act differently than when faced with lotteries concerning more strictly economic decisions. But until someone offers evidence or such an argument, the claim that risk aversion explains why increasing per capita income lowers civil war risk seems unsupportable.

So what does account for the strong empirical association between low per capita income and the propensity for civil war, at least over the last 60 years? Income is plausibly linked to two other variables in the model besides $y$, in ways that might help explain the regularity.

First, as seen above, more efficient counterinsurgency (larger $m$) implies fewer rebels and fewer soldiers in equilibrium. In the context of the model, government forces are more efficient when they capture or kill more rebels for given force sizes. Efficiency in this sense will be determined by factors such as the physical and social terrain, by government and rebel organizational capabilities, and by doctrine.

Richer countries are more urbanized and more uniformly covered by road and communication networks. These factors favor counterinsurgency and government control by making it harder for a nascent guerrilla band to hide. Small, highly secretive terrorist cells can operate in cities, but the movements and operations of larger rebel bands will be visible to many urban dwellers, and the opportunities for anonymous denunciation plentiful. In rural villages, where everyone knows everything about everybody, it is much easier to credibly threaten reprisal for denunciation or to hold small groups collectively responsible. Road networks reduce the amount of land that is good for hidden rebel camps and allow government forces to concentrate more quickly in response to

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28 More generally, constant relative risk aversion means that $-xu''(x)/u'(x)$ is constant.

29 In their textbook Mas-Colell, Whinston, and Green (1995, 192-3) write that “It is a common contention that wealthier people are willing to bear more risk than poorer people,” and that this is likely because richer people “can afford to take a chance.” Scott (1976) and in Political Science seems to agree. As noted in the introduction, assuming that there is a significant population that will probably starve to death unless they opt for and succeed at the rebel business could in principle explain the empirical regularity, but I think it is implausible that this is the source of many rebel bands in the last 60 years.
attacks.\textsuperscript{30}

Richer countries may also tend to have more efficient counterinsurgency due to better training, discipline, and possibly doctrine. Counterinsurgency is an extremely difficult political and military task. The main problems are acquiring the intelligence to distinguish reliably between active rebels and noncombatants, preventing military units from killing indiscriminately and so increasing support for the rebels, and preventing corruption in which military units loot and pillage from helpless populations. These problems may be more easily solved by a well-paid, well-trained, literate military with a strong chain of command and a strong sense of professionalism – all features that are plausibly correlated with per capita income.\textsuperscript{31} This is not to say that the problems are easily solved even then, as the experience of the US in Iraq and Vietnam, the USSR in Afghanistan, and the British in Northern Ireland testify. However, the comparison to counterinsurgency as practiced in Angola, Sudan, Liberia, Guatemala, El Salvador, or Peru – where government units regularly massacred, bombed or strafed whole villages suspected of collaborating with rebels – is instructive.

A second variable in the model that might vary systematically with income is $t$, the share of income that rebels and government can extract from locals. Suppose we slightly enrich the model by distinguishing between the rebel’s effective tax rate $t$, and the rate the government can get, call it $T$. These will often be different, since the government can tax cash crop production of the region by having the state marketing board pay farmers less than the world price (Bates 1981), or by controlling extractive industries in the region such as oil or mining. In richer countries where income is directly taxed, the government has the advantage of an extensive monitoring system. Also, income in a richer country is perhaps less subject to extortion by rebels because it is hypothetical money “kept” in banks, and because a large corporation’s decentralized structure makes it harder to threaten than a small business owner. In a richer economy with more income from human capital, individuals are more able to move in response to local extortion threats and the dangers of living in a conflict zone, making the effective tax rate for rebels lower. By contrast, farmers’ income comes from immovable capital.

With tax rates $t$ for the rebel group and $T$ for the government, the pure strategy equilibrium

\textsuperscript{30}Fearon and Laitin (2003) find that the percentage of mountainous terrain in a country is positively related to civil war risk, although this is not one of the most robust predictors. Empirically, road density and urbanization are related to lower civil war risk, but these are so closely correlated with per capita income that the effects are hard to distinguish. Kocher (2004) argues that the income-civil war relationship is mainly due to urbanization disfavoring rebellion. A suggestive set of examples comes from South America in the 1970s, where intellectuals (especially in Argentina) tried to develop urban-based insurgencies. They were in all cases quickly penetrated and destroyed by state militaries and secret services. On Argentina, see Gillespie (1982). That said, there are a few cases of successful urban insurgency, such as in Northern Ireland and now Iraq.

\textsuperscript{31}Felter (2005) uses extensive incident-level data from the Philippine military’s wars in Mindanao and Luzon to show that local units with highly trained leadership cadres are the most effective at killing or capturing rebels while avoiding civilian deaths.
in the game, when it exists, has

\[\alpha^* = \sqrt{\frac{c_g t}{c_r mT}} \quad \text{and} \quad \beta^* = \sqrt{\frac{T}{mc_G}} \left(\sqrt{\frac{\delta}{mc_G}} - \sqrt{\frac{c_R}{c_G mT}}\right).\]

Both government and rebel force sizes decrease as \( t \) falls, which, by the arguments above, I would expect to be the case for more economically developed countries.

6 What prevents a peaceful settlement?

Except in the case in which the government prefers to cede control of the region to the rebel group without a fight, the equilibrium outcome in the model is inefficient from the perspective of the rebel leadership and the government. Both sides could ideally do better. In the first place, the division of tax revenues that results from the violent interaction with force sizes \( \alpha^* \) and \( \beta^* \) could be replicated by an agreement in which the government chooses \( \beta = 0 \) and the rebels conscript just enough labor to collect from \( \delta \alpha^* \left(1 - p(\alpha^*, \beta^*)\right) \) nonrebels. In this way the government would save the cost of counterinsurgency and get a slightly greater extent of regional control. The rebels would save on the costs of rebellion, fewer people in the region would be diverted from productive to “appropriative” activities, and no one would be killed or captured.

Since the model assumes implicitly that the government can collect taxes in the region at zero marginal cost, an even more efficient arrangement would have the government control the whole region and no diversion of producers to insurgency or counterinsurgency (that is, \( \alpha = \beta = 0 \)). The government would transfer an amount to \( R \) at least equal to what \( R \) gets in the Nash equilibrium. This arrangement would minimize the inefficiency from the displacement of labor to “appropriative activities,” the traditional focus of contest models of conflict.

Alternatively, the government could agree to let the rebel group become the regional governmental authority, on the understanding that it would transfer tax revenues at least equal to what the government could obtain on net in the war equilibrium, though no more than what \( R \) could obtain in that equilibrium.\(^{32}\) Again, \( G \) would set \( \beta = 0 \).

Several minor modifications could introduce a larger and more realistic set of inefficiencies into the model. First, I assumed that individuals are taxed either by the rebel group or by the government, but not both. This will often be implausible. For example, peasants may grow crops for market that are taxed by government marketing agencies. If so, then they will be overtaxed by a logic like that in Shleifer and Vishny (1993) and Olson (1993), and consequently will produce less

\(^{32}\)An empirical example are the agreements between the Philippine government and the MNLF in 1993? that set up MNLF leader Nur Misuari as provincial leader. ...
than is optimal. Second, I assumed that the government is perfectly discriminate in its application
of force (no one but rebels are captured or killed), and I neglected that, over time, rebel and
government violence and threats destroy physical and human capital, discourage investment and
production, and encourage refugee flows.

In the one-shot version of the game, the problem with all the deals described above is that
they require one side to make a verbal commitment it would not want to carry out. If the gov-
ernment removed all military and police presence ($\beta = 0$), then the rebels would want to reneg
on any “spheres of influence” agreement by expanding to control the whole region. If the rebels
are granted control of the region in exchange for a division of tax revenues, the rebels have no
incentive to actually transfer these to the center if the center has no military threat against it. If the
government takes control promising to share revenues with $R$, it has no incentive to pay off the
rebel leadership if $R$ has no rebel force behind it.\footnote{\textsuperscript{33}}

In a repeated version of the interaction, we might in principle be able to construct an equi-
librium in which the two sides credibly commit to a mutually advantageous peace deal. I will
not explore this question thoroughly here, but will briefly indicate several significant obstacles to
enforcing a peace deal via reputation and conditional retaliation.

First, notice that $R$’s bargaining power depends on it remaining hidden. One obstacle to a
peace deal in which the government controls the region and pays off $R$ is that if the rebel leaders
become known to the government, the government could have an incentive to reneg, by capturing
or killing them. If, by agreeing to a peace deal, the rebel group brings itself into the open and so
exposes itself to a risk of being permanently eliminated or disadvantaged, then the mechanism of
enforcing cooperation by threatening conditional retaliation may be undermined.\footnote{\textsuperscript{34}}

Likewise, if the rebel group is to be given some measure of formal governmental author-
ity in the region, it will have to be allowed to gain enough military capacity that it can credibly
defend itself against government reneging (since it will be more exposed than before). But “too
much” military capacity, or the ability to secretly acquire more, would make the government con-
cerned about the rebel group reneging on the agreement by demanding yet more autonomy or even
secession.\footnote{\textsuperscript{35}}

\footnote{\textsuperscript{33}What about a deal in which both sides “arm” at levels $\alpha^*$ and $\beta^*$, but agree to spheres of influence in the region
and so avoid the costs of violence? The problem here is that having developed a force of $\beta^*$, it is in the government’s
interest to use them to take more control, for any division of revenues up to that implied by $\alpha^*$.}

\footnote{\textsuperscript{34}See Fearon (2004) for formal version of this argument. In that model, peace allows the government to recover
relative military strength and so can lead to reneging on a peace deal; as a result there may be no enforceable deal that
both sides prefer ex ante. Powell (2004) develops a theoretical generalization of how inefficient outcomes can occur
in repeated interaction where bargaining strength can permanently shift as a result of the interaction. In practice, one
way that rebel groups try to deal with this problem is by developing a public face in the form of a political party that
is ambiguously associated with the military arm of the group (e.g., Sinn Fein, or HB in Basque country).}

\footnote{\textsuperscript{35}No model of this dilemma exists, so far as I know, although Fearon (1996) considers a closely related problem in
A spheres-of-influence agreement in which the rebels control less than what they could given the government’s effort would seem to be the type of bargain most feasibly policed by an implicit understanding that violation would lead to more open and inefficient conflict. And in fact, “sitzkrieg” is a typical condition for rural guerrilla conflicts.\textsuperscript{36} Here, the rebels keep their arms and stay largely hidden, but restrain themselves from taking as much control as they could given the scale of government counterinsurgency in the short run. The government, on the other hand, does not conscript and employ the counterinsurgent force that would be short-run optimal given the size of the rebel force, understanding that if it did the rebels would scale up and the conflict would escalate to the $(\alpha^*, \beta^*)$ equilibrium level. If the government, by defecting to a major offensive, could permanently weaken the rebel force, then for the reasons given above this kind of relatively peaceful settlement could be rendered unenforceable.

To examine any of these commitment problem accounts using the model developed above, we would need to make assumptions concerning the replacement rate of captured or killed fighters in subsequent periods. Consider, for example, a two-period version of the game in which the regional population in the second period is reduced by the share of rebels captured or killed in the first period (that is, there is no natural population growth). If we maintain the assumption that per capita income does not change, then in the second period the size of the regional tax base is a bit smaller relative to the costs of conscription, making fighting less attractive for both $R$ and $G$. Under a “no replacement” assumption, then, the situation is clearly non-stationary and it is not obvious what the implications are for the prospects of a peace deal to be policed by implicit threats of retaliation. As an empirical matter, the leaders of counterinsurgency campaigns have repeatedly tried major offensives aimed at “breaking the back of the insurgency” in a single blow. This suggests that, realistically or not, they imagine that a sudden increase in the capture/kill rate can either eliminate the rebels or dry up their recruiting efforts for good.

Another hypothetically plausible type of explanation for inefficient fighting in some dynamic version of the model examined here would argue that one or both sides possess private information about their capabilities or resolve to fight, leading to the use of fighting as a costly but credible signal of capabilities or resolve.\textsuperscript{37} For example, the government might be unsure about the replacement rate of rebel fighters, which might be affected by rebel organizational capabilities and regional popular support for the rebel aims, both of which could be hard to observe directly. Such “war of attrition” explanations are frequently given in the media and sometimes by combatants themselves.\textsuperscript{38} They become increasingly less persuasive over time, however, especially for a five-bargaining between states.

\textsuperscript{36}See Keen (1998) on Liberia ... The Burmese government has likewise cut a series of deals with hill tribe rebel groups in which the rebels keep their arms and the two parties divide up revenues from the opium business (cites?).


\textsuperscript{38}Hamas e.g., others.
or ten- or even twenty-year fight that looks very much the same from year to year.

A final type of obstacle to a stable peace deal concerns the government fear that if it cuts a deal with one rebel group, it may soon face other insurgent groups, or splinter groups, making similar demands and employing guerrilla tactics to control territory. \( R \) may not have the capability to prevent new “entrants” from using the same technology of rebellion, which could be just as profitable after a peace deal between \( R \) and \( G \). The logic is that of the chain store paradox. The government prefers inefficient conflict in case A in order to deter inefficient conflict in cases B, C, D, and so on.\(^{39}\)

This mechanism would be expected to make for greater government intransigence in countries with a larger number of potentially secessionist minority groups. Walter (2005) empirically examines the relationship between the number of ethnic groups in a country and the government’s propensity to grant a measure of regional autonomy, and finds that this is lower in more diverse countries. The logic also suggests that peace deals will be more difficult to reach in conflicts where there is no dominant rebel organization capable of suppressing or controlling challengers.\(^{40}\)

7 Conclusion

The most common form of civil war in the post-World War II period has been a relatively small, stalemated guerrilla war confined to a rural periphery of a poor, post-colonial state. This is violence of a quite different sort from the French Revolution, the great model for theorizing about internal conflict. In the French Revolution paradigm, masses demonstrate in a capitol city, which leads to violent encounters with the coercive arms of the old regime. Such conflicts still occur, as in Iran and perhaps Nicaragua in 1978, or China (Tiananmen) and Romania in 1989. But this pattern is far more the exception than the rule. Nor is the model of the U.S. Civil War – essentially an interstate war fought by standing armies – at all common in this period.

I have argued that guerrilla warfare has distinct features and a distinct logic that, when incorporated in a model of the problem, help to explain its longevity and stability. In the French Revolution scenario, if the government is likely to collapse at a certain level of opposition, then there are “increasing returns” to adding more people to the opposition’s protests. As a technology

\(^{39}\)In the classic chain store paradox models, the equilibrium outcome is efficient because no entrant challenges and thus fighting is off the equilibrium path. To get inefficient fighting “on the path,” we would need to allow for heterogeneous types of entrants, some of which are willing to fight if they have zero regional control even if they face resistance, and whose preferences are not publicly observable. The explanation then becomes a private information story.

\(^{40}\)Peace negotiations for both Burundi and Somalia have been held up repeatedly by the appearance of new opposition groups demanding payoffs, just as an agreement is about to be signed.
for attempting to change the government, revolution is an all-or-nothing affair, a matter of tipping points, focal points, and successful or unsuccessful mass coordination.

By contrast, in the guerrilla war technology a relatively small number of poorly armed rebels survive by hiding successfully from government forces, which, if they knew where the rebels were, could fairly easily destroy them. Rough terrain can help hide the rebel groups, but preventing informers and denunciation to authorities is also essential. The problem of denunciation and detection, I have argued, implies that adding more rebels can increase the risks for existing fighters, and thus produce “decreasing returns” for the rebel movement (given a level of government resistance). This in turn leads to the possibility of a stable but violent equilibrium in which neither government nor rebels find it worthwhile to expand their efforts. Expansion would make the rebels too subject to detection and counterattack. A greater effort by the government would not yield enough new captures to make it worth the additional cost.

Higher per capita income means that there is in principle more stuff to appropriate if you are a member of a rebel band, and more stuff worth defending if you are on the government side. However, as argued above, the value of not fighting is also higher in a richer country, which raises the marginal costs of recruitment or conscription. These two considerations tend to offset each other, even when we consider risk aversion (and assuming that poorer people are not relatively more risk acceptant).

More plausible explanations for the empirical association between higher incomes and lower civil-war risk pose an indirect link, via the association of high income with (a) natural and social terrains inimical to guerrilla hiding, (b) possibly state military capability to conduct more efficient counterinsurgency, and (c) inability to appropriate as large a share of income through house-to-house visits by guerrillas, due in part to the mobility of human capital (as opposed to land).

At a theoretical level, the main innovation of the paper is the opening up of the traditional “contest success function” used to study an enormous variety of types of human and animal conflict. A natural next step would be to go further in this direction, developing a more explicit model of the information contest between rebels and government in this case of guerrilla war. In other words, can we “open up” the capture function used here, so providing plausible microfoundations for the assumption that adding rebels increases the risks of capture for all?
8 Appendix

In this appendix I analyze the model using a general capture function \( p(\alpha, m\beta) \) that satisfies certain conditions on its derivatives. A series of lemmas shows that under these conditions the best reply functions take the forms indicated in Figures 1 and 2. Uniqueness of pure strategy equilibrium is also demonstrated. The model analyzed in the text is a special case of this general model.

Define \( k(\alpha, \beta) = \alpha p(\alpha, m\beta) \), which is the number of rebels captured when the force sizes are \( \alpha \) and \( \beta \). Following the arguments in the text, we assume that \( p_1 \geq 0 \) and \( p_2 \geq 0 \), with the inequalities strict only if both \( \alpha \) and \( \beta \) are greater than zero. Thus \( k_1 \geq 0 \) and \( k_2 \geq 0 \), again with strict inequality only if both \( \alpha \) and \( \beta \) are positive.

After dividing out per capita income \( y \), the payoff functions (1) and (3) are

\[
\begin{align*}
    u_R(\alpha, \beta) &= t \min \{1 - \alpha, \delta \alpha - \delta k(\alpha, \beta)\} - c_R \alpha \quad \text{and} \\
    u_G(\alpha, \beta) &= t \max \{0, 1 - \alpha - \delta \alpha + \delta k(\alpha, \beta)\} - c_G \beta
\end{align*}
\]

I make the following assumptions about how the number captured varies with changes in forces sizes:

- \( k_{11} > 0 \) when \( \beta > 0 \): The number captured increases at an increasing rate for given (positive) government force size and increasing rebel force size. By the arguments about network connections and the strategy of hiding given in section 3, this is plausible, at least over the range where guerrilla tactics of hiding and hit-and-run attacks are necessary for the rebels. For instance, the assumption implies that adding more mafiosi increases the number prosecuted at an increasing rate. The capture model suggested in footnote 17 has this property as well.

- \( k_{22} < 0 \) when \( \alpha > 0 \): The number captured increases at a decreasing rate for a given rebel force and increasing numbers of police/soldiers. More soldiers or police are more likely to get good information about rebel unit whereabouts, or are more likely to encounter them at random, but the “returns” are diminishing. The capture model suggested in footnote 17 also has this property.

- \( k_{12} > 0 \): The effect of increasing government forces on the number captured is higher when the number of rebels is larger. Or likewise, the effect of increasing the number of active rebels on the number captured increases as there are more soldiers/police. (For instance, more FBI focused on the mafia means that adding more mafia becomes more dangerous for the mafia as a whole.) This is again true of the simple capture model suggested in footnote 17.

Lemma 1: Let \( \hat{\alpha} = 1/(1 + \delta) \). For \( \alpha < \hat{\alpha} \), \( G \)'s maximum constraint in (11) does not bind and \( G \)'s best reply given \( \alpha, \beta(\alpha) \), maximizes \( t(1 - \alpha - \delta \alpha) + t\delta k(\alpha, \beta) - c_G \beta \).
Proof. \(\alpha < \hat{\alpha}\) implies \(1 - \alpha - \delta\alpha > 0\). Since \(k(\alpha, \beta) \geq 0\), the max constraint in (11) does not bind.

Lemma 2. If

\[ t\delta k_2(\hat{\alpha}, 0) \leq c_G, \]  

then the unique Nash equilibrium is \((\alpha^*, \beta^*) = (\hat{\alpha}, 0)\).

Proof. By lemma 1, for \(\alpha \leq \hat{\alpha}\), \(\partial u_G / \partial \beta = t\delta k_2(\alpha, \beta) - c_G\). \(k_{22} < 0\) implies that this is decreasing in \(\beta\), so if (12) holds, then \(\partial u_G / \partial \beta < 0\) for all \(\beta \geq 0\). Thus \(G\) does best in this case to set \(\beta(\alpha) = 0\) for \(\alpha < \hat{\alpha}\).

For \(\alpha > \hat{\alpha}\), suppose to the contrary that there is an \(\alpha > \hat{\alpha}\) such that \(\beta' = \beta(\alpha) > 0\). \(k_{22} < 0\) and \(k(\hat{\alpha}, 0) = 0\) imply that \(t\delta k(\hat{\alpha}, \beta') < \beta' t\delta k_2(\hat{\alpha}, 0)\). Subtracting \(c_G \beta'\) from both sides and rewriting we have,

\[ t\delta k(\hat{\alpha}, \beta') - c_G \beta' < \beta' (t\delta k_2(\hat{\alpha}, 0) - c_G). \]

The right-hand side is less than or equal to zero by (12), so the left-hand side is strictly less than zero. But then \(t(1 - \alpha - \alpha \delta) + t\delta k(\hat{\alpha}, \beta') - c_G \beta' < 0\), which implies that the max condition in (11) binds. Inspection of (11) shows that when the max condition binds, the optimal \(\beta(\alpha) = 0\), which contradicts \(\beta' > 0\).

Thus (11) implies that \(\beta(\alpha) = 0\) for all \(\alpha\). Since this is true for each \(\alpha\), \(G\)'s best reply will also be zero for any mixture by \(R\) over different values of \(\alpha\). So (11) implies \(\beta^* = 0\), and therefore Nash equilibrium requires that \(R\) play its best reply to \(\beta^* = 0\), which is \(\hat{\alpha}\).

Lemma 3. If \(t\delta k_2(\hat{\alpha}, 0) > c_G\), then there are \(\alpha\) and \(\hat{\alpha}\) such that \(0 < \alpha < \hat{\alpha} < \bar{\alpha}\), and \(\beta(\alpha) > 0\) and \(\beta'(\alpha) > 0\) for \(\alpha \in (\alpha, \hat{\alpha})\) and \(\beta(\alpha) = 0\) otherwise. At \(\alpha = \bar{\alpha}\), \(\beta(\alpha)\) has two values, one strictly positive and one zero.

Proof. By lemma 1, for \(\alpha < \hat{\alpha}\), \(\beta(\alpha)\) maximizes \(t(1 - \alpha - \alpha \delta) + t\delta k(\hat{\alpha}, \beta) - c_G \beta\). So, either \(t\delta k_2(\alpha, \beta(\alpha)) = c_G\), or if \(t\delta k_2(\alpha, 0) < c_G\) then \(\beta(\alpha) = 0\) (since \(k_{22} < 0\)). Since \(k_2(0, 0) = 0\) the latter certainly obtains for \(\alpha = 0\). \(k_{12} > 0\) and \(t\delta k_2(\hat{\alpha}, 0) > c_G\) imply that there is a unique \(\bar{\alpha}\) that solves \(t\delta k_2(\bar{\alpha}, 0) = c_G\), and that \(\bar{\alpha} < \hat{\alpha}\). By the first order conditions, for \(\alpha < \bar{\alpha}\), \(G\) does best to set \(\beta(\alpha) = 0\), and for \(\alpha > \bar{\alpha}\), \(G\) chooses the unique \(\beta(\alpha)\) that satisfies \(t\delta k_2(\alpha, \beta(\alpha)) = c_G\) (unique because \(k_{22} < 0\)), provided the max constraint does not bind.

Differentiating \(t\delta k_2(\alpha, \beta(\alpha)) = c_G\) implicitly in \(\alpha\) and solving yields \(\beta'(\alpha) = -k_{12} / k_{22}\), which is positive if there are diminishing returns to counterinsurgency and \(k_{12} > 0\). Thus \(\beta(\alpha)\) slopes upwards for \(\alpha > \bar{\alpha}\) and when the max constraint does not bind.
As long as the max constraint does not bind, $G$’s equilibrium payoff is decreasing in $\alpha$:

$$\frac{\partial}{\partial \alpha} t(1 - \alpha - \delta\alpha - \delta k(\alpha, \beta(\alpha))) - c_G\beta(\alpha) < 0$$

which is true since the left-hand side is zero and the right-hand side is greater than zero by $k_1 < (1 + \delta)/\delta$, which is shown in Lemma X below. $u_G(1, \beta)$ is necessarily negative for $\beta > 0$, and $G$ gets a strictly positive payoff when $\alpha = \tilde{\alpha}$, since $u_G(\tilde{\alpha}, 0) = 0$ and $t\delta k_2(\tilde{\alpha}, 0) > c_G$ means that $u_G(\tilde{\alpha}, \beta)$ is increasing in $\beta$ at $\beta = 0$. Thus there exists a unique $\tilde{\alpha} > \hat{\alpha}$ such that $u_G(\tilde{\alpha}, \beta(\tilde{\alpha})) = 0$. Above this level, $\beta(\alpha) = 0$ since the $\beta$ that solves $t\delta k_2(\alpha, \beta) = c_G$ must yield negative profits.\(^{41}\)

At $\alpha = \tilde{\alpha}$, get zero profits for either ...

\[\square\]

**Lemma 4.** Define $u_R^u(\alpha, \beta) = (t\delta - c_R)\alpha - t\delta k(\alpha, \beta)$, which is the rebel group’s utility function ignoring the constraint.\(^{42}\) Define $\alpha_u(\beta)$ as the $\alpha \in [0, 1]$ that maximizes this for given $\beta$. Define $\beta$ as the unique solution to $k_1(1, \beta) = 1 - c_R/t\delta$ if a solution exists, and set $\beta = \infty$ otherwise. Define $\beta$ as the solution to $k_1(0, \beta) = 1 - c_R/t\delta$ if it exists and infinity otherwise. Then:

(i) If $\beta = \infty$, $\alpha_u(\beta) = 1$ for all $\beta$.

(ii) If $\beta$ is finite, then $0 < \tilde{\beta} < \beta$ and $\alpha_u(\beta) = 1$ for $\beta \leq \tilde{\beta}$, $\alpha_u(\beta)$ is strictly decreasing from 1 to 0 for $\beta \in [\tilde{\beta}, \beta]$, and $\alpha_u(\beta) = 0$ for $\beta \geq \tilde{\beta}$ if $\beta$ is finite.

**Proof.** For (i), note first that $u_R^u$ increases in $\alpha$ so long as $\partial u_R^u/\partial \alpha = t\delta - c_R - t\delta k_1(\alpha, \beta) > 0$, or $k_1(\alpha, \beta) < 1 - c_R/t\delta$. Because $k_1(1, 0) = 0$, $\beta$ fails to exist when $k_1(1, \beta) < 1 - c_R/t\delta$ for all $\beta$. So in this case $\alpha_u(\beta) = 1$ for all $\beta$.

For (ii), $k_1(0, 0) = 0$ and $k_{12} > 0$, so either there is a unique $\tilde{\beta}$ such that $k_1(0, \tilde{\beta}) = 1 - c_R/t\delta$ or $\beta$ can grow arbitrarily large and still $u_R^u$ is increased by increasing $\alpha$ above zero. In the former case, $k_{12} > 0$ implies that $k_1(0, \beta) \geq 1 - c_R/t\delta$ for $\beta \geq \tilde{\beta}$, and thus $\alpha_u(\beta) = 0$.

When $\beta$ is finite, by definitions, $k_1(1, \beta) = 1 - c_R/t\delta = k_1(0, \beta)$, which implies that $\tilde{\beta} < \beta$ (again using $k_{12} > 0$). Whether $\beta$ is finite or not, for $\beta \in (\tilde{\beta}, \beta)$, $k_1(0, \beta) < 1 - c_R/t\delta < k_1(1, \beta)$. Thus,

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\(^{41}\)Proof that $\hat{\alpha} < \tilde{\alpha}$ that doesn’t use decreasing $u_G$: First, note that for all $\alpha \leq \hat{\alpha}$, $G$ gets strictly positive profits when it chooses it best reply $\beta(\alpha)$. The max constraint does not bind for $\alpha \leq \hat{\alpha}$, which implies $u_G(\alpha, \beta) = t(1 - \alpha - \delta\alpha + t\delta k(\alpha, \beta) - c_G\beta)$. This is necessarily positive for $\alpha < \hat{\alpha}$ and $\beta(\alpha) = 0$, and also for $\alpha \in [\hat{\alpha}, \tilde{\alpha}]$ since it is non-negative at $(\alpha, 0)$ and increasing in $\beta$ by $t\delta k_2(\alpha, 0) > c_G$ for $\alpha \in [\hat{\alpha}, \tilde{\alpha}]$. Now suppose to the contrary that $\hat{\alpha} < \tilde{\alpha}$. This contradicts the $u_G(\tilde{\alpha}, \beta(\tilde{\alpha})) = 0$, which is the definition of $\tilde{\alpha}$. Thus $\hat{\alpha} < \tilde{\alpha}$.

\(^{42}\)Recall that we assume $t\delta > c_R$, which means that the rebel group would want to take and control territory if there were no central government response.
by \( k_{11} > 0 \) there is a unique \( \alpha \) such that \( k_1(\alpha, \beta) = 1 - c_R/t\delta \), which implies that this \( \alpha = \alpha_u(\beta) \) since this is the first-order condition for \( u_R^k \), and the second derivative is negative by \( k_{11} > 0 \).

Implicitly differentiating the first-order condition in \( \beta \) and rearranging shows that \( \alpha_u'(\beta) = -k_{12}/k_{11} \), which is negative, as claimed. Finally, for \( \beta < \tilde{\beta} \), \( k_1(1, \beta) < k_1(1, \tilde{\beta}) = 1 - c_R/t\delta \) implies \( \alpha_u(\beta) = 1 \) for these \( \beta \). \( \square \)

**Lemma 5.** When the constraint in (10) binds, \( \alpha(\beta) \) solves \( 1 - \alpha = \delta\alpha - \delta k(\alpha, \beta) \), and \( \alpha'(\beta) > 0 \). Further, the constraint binds at \( \beta = 0 \), with \( \alpha(0) = \hat{\alpha} \).

**Proof.** The constraint in (10) binds when

\[
1 - \alpha_u(\beta) < \delta\alpha_u(\beta) - \delta k(\alpha_u(\beta), \beta).
\]  

\( \delta\alpha - \delta k(\alpha, \beta) \) is zero at \( \alpha = 0 \) and increases up to its maximum at a point which is greater than \( \alpha_u(\beta) \), since the first order condition is \( k_1(\alpha, \beta) = 1 \) (which requires that the solution be greater than \( \alpha_u(\beta) \) since \( \alpha_u(\beta) \) solves \( k_1(\alpha, \beta) = 1 - c_R/t\delta \)). Thus there is a unique \( \alpha \) that solves \( 1 - \alpha = \delta\alpha - \delta k(\alpha, \beta) \). This \( \alpha \) is optimal for \( R \), since any smaller force size would mean that the constraint was not binding yet \( u_R^u(\alpha, \beta) \) would be increasing. Any greater \( \alpha \) would reduce \( R \)'s payoff by reducing \( 1 - \alpha \) and increasing the cost \( c_R\alpha \).

\( \alpha'(\beta) > 0 \) follows by differentiating \( 1 - \alpha = \delta\alpha - \delta k(\alpha, \beta) \) implicitly in \( \beta \) and using \( k_2 > 0 \) and \( k_1 < 1 \). The latter holds because for \( \alpha < \alpha_u(\beta), k_1(\alpha, \beta) < 1 - c_R/t\delta < 1 \).

At \( \beta = 0 \), \( u_R = t \min\{1 - \alpha, \delta\alpha\} - c_R\alpha \), which is maximized at \( \alpha = \hat{\alpha} \). \( \square \)

**Lemma 6.** \( R \)'s best reply function \( \alpha(\beta) \) either (a) increases from \( \alpha(0) = \hat{\alpha} \) for all \( \beta > 0 \), or (b) increases from \( \alpha(0) = \hat{\alpha} \) up to a maximum at a \( \tilde{\beta} > 0 \), and then decreases, either continuously or to zero for all \( \beta \geq \tilde{\beta} \) if \( \beta \) is finite.

**Proof.** The constraint in (10) binds when (13) obtains. Because \( \alpha_u(\beta) = 1 \) for \( \beta < \tilde{\beta} \) and because \( k_2 > 0 \), this constraint certainly holds for \( \beta \in [0, \tilde{\beta}] \). Thus \( \alpha(\beta) \) is the increasing function defined in Lemma 5 for \( \beta \) in this interval. For \( \beta > \tilde{\beta} \), there may exist a \( \hat{\beta} \) such that \( 1 - \alpha_u(\hat{\beta}) = \delta\alpha_u(\hat{\beta}) - \delta k(\alpha_u(\hat{\beta}), \beta) \), in which case the constraint no longer binds for \( \beta > \hat{\beta} \), and the decreasing function \( \alpha_u(\beta) \) becomes the best reply function for these \( \beta \). (Note that the left hand side of the inequality is nondecreasing in \( \beta \) and strictly positive for \( \beta \geq \tilde{\beta} \), while the right-hand side is strictly decreasing for \( \beta < \tilde{\beta} \). Thus if it exists an intersection is unique, and for \( \beta > \beta \) the constraint no longer binds.)
If there is no intersection (which is possible only if $\beta = \infty$), then (13) holds for all $\beta$, implying that the constraint binds and $\alpha(\beta)$ is given as in Lemma 5.

**Lemma 7.** There is no pure strategy Nash equilibrium in which the constraint binds and $\beta^* > 0$.

**Proof.** Suppose to the contrary that there is a pair $(\alpha^*, \beta^*)$ with $\beta^* > 0$ such that $1 - \alpha^* = \delta \alpha^* - \delta k(\alpha^*, \beta^*)$ (which is to say that the rebels choose the force size that gives them control of all noncombatants in the region, given the government force size $\beta^*$). Rewriting yields

\[ 1 - \alpha^* - \delta \alpha^* + \delta k(\alpha^*, \beta^*) = 0 \]
\[ t(1 - \alpha^* - \delta \alpha^*) + t\delta k(\alpha^*, \beta^*) - \beta^* c_G < 0. \]

Thus if the constraint binds and $G$ chooses to deploy forces, $u_G < 0$, which is impossible in equilibrium since $G$ can guarantee itself at least zero by setting $\beta^* = 0$.

**Lemma 8.** If $t\delta k_2(\hat{\alpha}, 0) > c_G$, then either there is no pure strategy equilibrium or there is a unique pure strategy equilibrium.

**Proof.** There is clearly no pure strategy equilibrium in which $\beta^* = 0$ since the condition implies that $G$ can increase its payoff with a strictly positive $\beta$ against $\hat{\alpha}$. Lemma 7 implies that if there is a pure strategy equilibrium in this case, it must involve an intersection of the curves $\beta(\alpha)$ and $\alpha(\beta)$ defined implicitly by $t\delta k_2(\alpha, \beta(\alpha)) = c_G$ for $\alpha \in [\bar{\alpha}, \alpha]$ and $k_1(\alpha(\beta), \beta) = 1 - c_R/t\delta$ for $\beta \in (\hat{\beta}, \bar{\beta})$. Since in these regions $\beta'(\alpha) > 0$ and $\alpha'(\beta) < 0$ by lemmas 3 and 5, if they intersect the intersection is unique.
References


