

Regime Change and Revolutionary Entrepreneurs*

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Preliminary. Comments Welcome.

Abstract

I develop a model of regime change to study how a revolutionary vanguard may use violence to coordinate and mobilize members of a mass public by convincing them that the level of anti-government sentiment in society is high. The model is consistent with unexpectedly successful violence by vanguards sparking revolution, but also suggests that the relationship between vanguards and the micro-foundations of revolution is complicated and subtle. The model has multiple equilibria, some where revolution is relatively likely and some where it is relative unlikely or impossible. Within an equilibrium, structural factors affect the likelihood of revolution. Nonetheless, if two identical societies play different equilibria, they have very different likelihoods of experiencing revolution, creating a problem for empirical work seeking to identify the root causes of political violence. This fact also suggests that standard empirical arguments about the importance of revolutionary vanguards may be problematic. In equilibrium, there are selection effects—even controlling for all relevant structural factors, vanguards arise only in those societies that are particularly likely to have successful regime change even in the absence of a vanguard. Finally, on the purely theoretical side, the model studied here is closely related to a global game, yet generates multiple equilibria regardless of informational assumptions. This multiplicity result helps to clarify the substantive content of the technical assumptions underlying uniqueness results in the applied literature on global games of regime change.

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I explore how violence by a revolutionary vanguard may facilitate coordination and mobilization of mass publics for the purpose of effecting regime change. The model highlights a mechanism by which revolutionary vanguards may achieve such coordination and mobilization. They use violence to convince the public that there is a high level of anti-government sentiment in society. However, it also suggests ways in which standard empirical arguments about the importance of revolutionary vanguards may be problematic. In equilibrium, there are selection effects—even controlling for all relevant structural factors, vanguards will arise only in those societies that are particularly likely to have successful regime change even in the absence of a vanguard. Hence, a correlation between active revolutionary vanguards and violent regime change may not constitute evidence for the causal importance of vanguards.

I develop a model of revolution with a prior stage in which a revolutionary entrepreneur engages in publicly observable political violence. In the model, successful regime change occurs only if a sufficient number of individuals participate. Individuals know their own level of anti-government sentiment (and thus how much they would benefit from regime change), but are uncertain of how anti-government their fellow citizens are. Individual levels of anti-government sentiment are positively correlated, so an individual's personal beliefs provide some information about the distribution of beliefs in society.

This situation creates a coordination problem—citizens of a given type want to participate in the revolution if and only if they believe enough other citizens will also participate. Revolutionary entrepreneurs use violence to convince citizens that the level of anti-government sentiment is high, so that they believe many of their fellow citizens will participate.

Violence is an effective tool for communicating the level of anti-government sentiment in society because the ability of the revolutionary vanguard (e.g., terrorists or guerillas) to produce attacks is a function of both their own costly effort and of their support in the public at large. Insurgents often depend on members of the population for material support, safe havens, information, and recruits. Moreover, members of the population can betray insurgents to the government. Thus, even though the revolutionary vanguard has no private information about public sentiment, successful acts of violence suggest higher levels of anti-government sentiment. (See Kalyvas (1999) for a discussion of the relationship between insurgents and populations.)

The primary contribution of the paper is in helping to provide theoretical micro-foundations for the existence of a credible revolutionary threat. The model shows a mechanism by which violence by a vanguard may help to spark a successful revolution. Such a theory is important for at least two reasons. First, violent regime change is itself a political outcome of considerable interest. Second, the revolutionary threat plays a major role in political economy explanations of a variety of important phenomena, including democratization, redistribution, regime stability, the organization of the state, corruption, and public goods provision (see, for example, Acemoglu and Robinson 2001, 2006; Bueno de Mesquita et al. 2003; Bueno de Mesquita and Smith 2008; Fearon 2006; Myerson 2007).

I also contribute to ongoing debates regarding the origins of revolution and violent regime

change. A common critique of purely structural accounts of violent regime change (i.e., accounts based purely on structural characteristics of a society—the economy, international threats, regime capacity, etc.—to the exclusion of strategic behavior and other factors) is that the structural conditions characterizing societies that experience revolution often also characterize societies that do not experience revolution.

The model presented here calls into question the validity of this empirical critique and its policy implications. The model has multiple equilibria—some where revolution is relatively likely and some where it is relatively unlikely or impossible. Within an equilibrium, comparative statics show that structural factors affect the likelihood of revolution. Nonetheless, if two structurally identical societies play different equilibria, they have very different likelihoods of experiencing revolution.

This argument suggests a quite general problem both for the empirical literature on the root causes of political violence and for policymaking. In a world characterized by multiple equilibria, much of the variation in the data may be due to whatever second-order factors determine equilibrium selection, rather than those structural factors that we often think are of first-order importance for explaining political violence. Thus, structural factors may matter (for a given equilibrium selection) but be difficult to detect empirically because we cannot observe which equilibrium a society is playing. Moreover, from the perspective of policymaking, this implies that, even though the data are not well explained by structural variation, it may well be that, within a given society (playing its particular equilibrium), changing key structural factors would reduce political violence or the likelihood of violent regime change.

Critics of structural accounts further argue that a key additional explanatory variable is the presence of a revolutionary vanguard (DeNardo 1985; Popkin 1988; Kurrild-Klitgaard 1997). Structurally ripe societies, the argument goes, have revolutions only if the right type of revolutionary leadership arises. Proponents of this argument point to a variety of examples of small vanguards engaging in violence that appears to have inspired a larger insurrection. For instance, the FLN's (National Liberation Front) terrorist campaign helped spark the Algerian War of Independence (Kalyvas 1999). Violence by Argentine guerilla groups such as the Montoneros and the ERP (People's Revolutionary Army) in the late 1960s and early 1970s led to much larger scale insurgency by the mid-1970s (Gillespie 1995). And terrorist tactics and other forms of violent agitation by Russian revolutionaries helped set the stage for the 'spontaneous' uprisings of 1905 and 1917 (DeNardo 1985).

My model is certainly supportive of the idea that successful vanguard violence can spark large-scale uprisings. However, it also calls into question the logic underlying this empirical argument. In equilibrium, the vanguard only engages in high levels of violence if society is playing the equilibrium where revolution is relatively likely. This fact suggests that there may be selection effects that need to be accounted for before making inferences from empirical observation. In particular, a society with an active vanguard will be more likely to have a revolution (all else equal), even if the vanguard itself has no effect, because active vanguards only emerge in those societies playing equilibria in which revolution is relatively likely.

The model has a few additional substantive implications. I provide comparative statics on how factors such as regime capacity and the structure of incentives inside the revolutionary organization affect the likelihood of successful revolution. The model also predicts that participants in revolutions not fomented by an active revolutionary vanguard are expected to be more anti-government than other revolutionaries.

In addition to these substantive implications, the equilibrium characterization may be of some independent interest for the applied theory literature on global games of regime change. A key result in that literature shows that, in coordination games, when players have heterogeneous information and private information is sufficiently informative, there is a unique equilibrium (Angeletos, Hellwig and Pavan 2006, 2007; Guimaraes and Morris 2007; Edmond 2007). This uniqueness result has made global games of regime change workhorse models for a variety of applications. My model is not a global game, but its second stage is closely related to this literature, yet generates multiple equilibria, even when treated as a stand-alone game with exogenous information. An exploration of this fact yields some insight into the sources of uniqueness in global games of regime change and the verisimilitude of the assumptions needed to generate such uniqueness.

The paper proceeds as follows. Section 1 describes the model and Section 2 defines and characterizes equilibrium. Section 3 discusses substantive implications for regime change and relates the mechanism for coordinating and mobilizing mass publics modeled here to other mechanisms discussed in the political economy literature. Section 4 explains my multiplicity result and compares it to the standard uniqueness results for global games of regime change. Section 5 concludes.

1 A Model of Revolutionary Entrepreneurs and Regime Change

There are a revolutionary vanguard and a continuum of population members of measure 1. At the beginning of the game, each member of the population i learns her type $\theta_i = \theta + \epsilon_i$. The common component θ is drawn by Nature from a normal distribution with mean m and variance σ_θ^2 . The idiosyncratic components ϵ_i are independent draws by Nature from a normal distribution with mean 0 and variance σ_ϵ^2 . Members of the population observe only θ_i , not θ or ϵ_i . After each population member observes her type, the vanguard, which also does not know θ , chooses a level of effort to expend on a campaign of violence (e.g., terrorist or guerrilla attacks). Individuals observe the level of violence and then decide whether or not to join a revolution against the government. The game ends with the government either being overthrown or remaining in place.

I refer to the stage of the game in which players decide whether or not to participate in the revolution as the “revolution stage” and the stage where the revolutionary entrepreneurs engage in violence as the “vanguard stage”.

The measure of people who join the revolution is N . The regime is replaced if and only if N is greater than or equal to a threshold $T \in (0, 1)$.

A member of the population, i , chooses $a_i \in \{0, 1\}$, where $a_i = 1$ is the decision to participate. A person’s type, θ_i , determines how much she values regime change. She derives a portion, $1 - \gamma \in$

$(0, 1)$, of that value whether or not she personally participates in the revolution. The other portion, γ , is realized only if the revolution succeeds and the individual participated in it. The payoff to a failed revolution is normalized to 0. Participating imposes a cost $k > 0$ on the individual. The payoffs for members of the population are given by the following von Neumann-Morgenstern expected utility function:

$$U_i(a_i, \theta_i, \gamma, k, N, T) = \begin{cases} (1 - (1 - a_i)\gamma)\theta_i - a_i k & \text{if } N \geq T \\ -a_i k & \text{if } N < T. \end{cases}$$

These payoffs are most easily understood from the following payoff matrix for a representative player i .

		$N < T$	$N \geq T$
Player i	$a_i = 0$	0	$(1 - \gamma)\theta_i$
	$a_i = 1$	$-k$	$\theta_i - k$

Payoffs for a Representative Player i

Denote by $t \in [\underline{t}, \infty)$ (with $\underline{t} \geq 0$) the level of effort exerted by the revolutionary vanguard. The total level of violence observed by the population prior to the decision of whether or not to join the revolution is $v = t + \theta + \eta$, where η is drawn by Nature from a normal distribution with mean 0 and variance σ_η^2 . As discussed in the introduction, the idea is that when the public has a higher level of anti-government sentiment, it is easier for the revolutionary vanguard to produce violence, since it will have the support of the population. The vanguard benefits from a successful revolution and bears costs for effort. The vanguard's payoffs are given by the following von Neuman-Morgenstern expected utility function:

$$U_v(t, N, T) = \begin{cases} 1 - c(t) & \text{if } N \geq T \\ -c(t) & \text{if } N < T, \end{cases}$$

where $c' > 0$, $c'' > 0$, $c''' \geq 0$, and c' satisfies $\lim_{t \rightarrow \underline{t}} c'(t) = 0$ and $\lim_{t \rightarrow \infty} c'(t) = \infty$.¹

1.1 A Comment on Payoffs

There are two key assumption on payoffs.

¹As will be suggested by Lemma 11, slightly more structure is needed on the cost function to guarantee existence of a pure strategy equilibrium. In particular, for some finite, positive \bar{c} , I need $c''(0) \geq \bar{c}$. The value of \bar{c} is suggested by the second-order condition for the vanguard's optimization problem, but I have not yet characterized it in terms of primitives. The idea is that the convexity in c needs to compensate for any convexity that enters into the vanguard's objective through the normal cdf, so that the vanguard's objective is concave.

First, there is some portion (γ) of the payoffs from revolutionary success that can only be accessed by those who participate in the revolution. As a matter of verisimilitude, the assumption seems reasonable. In many settings, those who actively participate in revolution gain privileged status either within the government or within society after regime change occurs.

It is worth noting that this assumption relaxes the standard collective action problem, since there is the possibility of a private benefit to participation if the revolution succeeds. This fact is consistent with Kalyvas’s (2007) argument that the collective action problem may be less binding in violent mobilization than most rational choice scholarship suggests. Here, fairly natural assumptions on payoffs have the effect of relaxing the collective action problem and focusing instead on coordination.

Second, there is heterogeneity in the level of anti-government sentiment, but population members’ views are positively correlated. The idea here is that particularly bad (resp. good) governments are likely, on average, to generate more (resp. less) anti-government sentiment.

It is also worth pointing out that, although the revolution stage of this model is similar to models from the literature on global games of regime change (Angeletos, Hellwig and Pavan 2006, 2007; Edmond 2007), it is not a global game. In particular, unlike in those earlier papers, the model here does not satisfy the limit dominance property of global games (Morris and Shin 1998). In particular, there is no θ_i such that participation is a dominant strategy.

2 Equilibrium

A pure strategy for the vanguard is a choice, t , of effort directed at violence. A pure strategy for a member of the population is a mapping $a_i(\theta_i, v) : \mathbb{R} \times \mathbb{R} \rightarrow \{0, 1\}$, from observed levels of personal anti-government sentiment and vanguard violence into a decision of whether or not to participate.

The solution concept is pure strategy Perfect Bayesian Equilibrium (PBE). Since, for any choice t , there is no state θ that is inconsistent with any observation v or θ_i , no event is off the path. Thus, the solution concept simply requires that beliefs be consistent with the strategy profile and Bayes’ rule.

I restrict the set of equilibria in two ways. First, I restrict attention to those pure strategy equilibria of the full game in which, in the game that constitutes the revolution stage, players use simple cutoff strategies of the form, “choose $a_i = 1$ if and only if $\theta_i \geq \hat{\theta}(v)$.”

Second, for some realizations of v , the game that constitutes the revolution stage turns out to have multiple equilibria in cutoff strategies: one with an infinite cutoff (i.e., no participation) and two with finite cutoffs. I focus on equilibria of the full game in which players play the same selection—i.e., the low cutoff, the high cutoff, or the infinite cutoff—whenever there are multiple equilibria. This can be thought of as imposing a kind of continuity on the population members’ strategy—small changes in v do not result in large changes in the cutoff rule by changing the equilibrium selection.

The idea behind this second requirement is twofold. First, which equilibrium is played when

multiple equilibria exist is likely to be a fact about the culture and society in question. It would be strange if small changes in the level of revolutionary violence could change people's expectations about the fundamental conjectures others in their society have about each other's behavior. Second, the purpose of the model is to explore how revolutionary violence can affect mobilization. It would be stacking the deck in favor of finding a large effect to assume that small changes in revolutionary violence could change mobilization by changing the type of equilibrium that is played. Instead, I will exam how violence affects mobilization by shifting the cutoff rule, holding fixed the selection of a cutoff rule from among those that are consistent with equilibrium.

I refer to a pure strategy PBE that satisfies these two criteria as a *cutoff equilibrium*.

2.1 Beliefs

A person of type θ_i , after observing her type but not the level of violence v , has posterior beliefs about θ that are distributed normally with mean

$$\bar{m}_i = \lambda\theta_i + (1 - \lambda)m$$

and variance

$$\sigma_1^2 = \lambda\sigma_\epsilon^2,$$

with

$$\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}.$$
²

Suppose that the common belief is that the level of effort by the revolutionary vanguard was t^* . Then the members of the population believe that $v - t^*$ is a mean θ normally distributed random variable with variance $\sigma_\theta^2 + \sigma_\eta^2$. After observing a level of violence, a person of type θ_i has posterior beliefs about θ that are normally distributed with mean

$$\bar{\bar{m}}_i = \psi(v - t^*) + (1 - \psi)\bar{m}$$

and variance

$$\sigma_2^2 = \psi(\sigma_\theta^2 + \sigma_\eta^2)$$

with

$$\psi = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_\theta^2 + \sigma_\eta^2}.$$

2.2 The Revolution Stage

Since all members of the population are measure zero, an individual participates if and only if:

$$\Pr(N \geq T|\theta_i, v)\theta_i - k \geq \Pr(N \geq T|\theta_i, v)(1 - \gamma)\theta_i$$

²The updating follows from a standard application of Bayes' Rule with normally distributed priors and signals (DeGroot 1970).

or

$$\Pr(N \geq T | \theta_i, v) \gamma \theta_i \geq k. \quad (1)$$

For any beliefs, there is always an equilibrium of the revolution stage where no one participates, regardless of type (i.e., with an infinite cutoff rule).

Lemma 1 *For all parameter values, there is an equilibrium of the game characterizing the revolution stage in which no player participates.*

All proofs are in Appendix A.

I now look for pure strategy equilibria in cutoff strategies in the revolution stage with positive participation. Fix a belief t^* and a realization of v and suppose there is such a cutoff rule, $\hat{\theta}(v - t^*) < \infty$. Given a realization of θ , a person will participate if $\epsilon_i \geq \hat{\theta}(v - t^*) - \theta$. The level of participation will be $N(\theta, \hat{\theta}(v - t^*)) = 1 - \Phi\left(\frac{\hat{\theta}(v - t^*) - \theta}{\sigma_\epsilon}\right)$, where Φ is the cumulative distribution function of the standard normal. Victory will be achieved if $N(\theta, \hat{\theta}(v - t^*)) \geq T$. For a fixed $\hat{\theta}(v - t^*)$, $N(\theta, \hat{\theta}(v - t^*))$ is strictly increasing in its first argument—the more anti-government sentiment, the greater participation. Thus, for a given cutoff rule and a fixed $v - t^*$, the minimal level of anti-government sentiment necessary for a successful anti-government campaign is $\theta^*(\hat{\theta}(v - t^*))$, defined by:

$$N(\theta^*, \hat{\theta}(v - t^*)) = 1 - \Phi\left(\frac{\hat{\theta}(v - t^*) - \theta^*}{\sigma_\epsilon}\right) = T.$$

or

$$\theta^*(\hat{\theta}(v - t^*)) = \hat{\theta}(v - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon. \quad (2)$$

From the perspective of a member of the population of type θ_i , given a cutoff rule $\hat{\theta}(v - t^*)$, the probability of victory is

$$\Pr(\theta \geq \theta^*(\hat{\theta}(v - t^*)) | \theta_i, v) = 1 - \Phi\left(\frac{\theta^*(\hat{\theta}(v - t^*)) - \bar{m}_i}{\sigma_2}\right).$$

From Equation 1, such a person will choose to participate if

$$\left[1 - \Phi\left(\frac{\theta^*(\hat{\theta}(v - t^*)) - \bar{m}_i}{\sigma_2}\right)\right] \gamma \theta_i \geq k.$$

Since \bar{m}_i is increasing in \bar{m}_i , which is increasing in θ_i , the left-hand side of this inequality is increasing in θ_i . This monotonicity implies that if a person with type $\theta_i = \hat{\theta}(v - t^*)$ participates, then so will a person with $\theta_i > \hat{\theta}(v - t^*)$. Substituting for \bar{m}_i and \bar{m}_i , if such a cutoff rule exists, for a given $v - t^*$, it is given by:

$$\left[1 - \Phi\left(\frac{\theta^*(\hat{\theta}) - \psi(v - t^*) - (1 - \psi)\lambda\hat{\theta} - (1 - \psi)(1 - \lambda)m}{\sigma_2}\right)\right] \gamma \hat{\theta} = k.$$

A function, $\hat{\theta}(\cdot)$, giving equilibrium cutoff strategies must satisfy this equality and Equation 2. Substituting from Equation 2, it is possible to implicitly define a function $\hat{\theta}(\cdot)$ that satisfies both these requirements in one condition:

$$\left[1 - \Phi \left(\frac{(1 - (1 - \psi)\lambda)\hat{\theta} - \frac{(1 - \psi)(1 - \lambda)m + \sigma_\epsilon \Phi^{-1}(1 - T) + \psi(v - t^*)}{\sigma_2}}{\sigma_2} \right) \right] \gamma \hat{\theta} = k \quad (3)$$

It will be useful to define the following:

$$f(x, v - t^*) \equiv \alpha x - \beta,$$

where $\alpha = \frac{(1 - (1 - \psi)\lambda)}{\sigma_2}$ and $\beta = \frac{(1 - \psi)(1 - \lambda)m + \sigma_\epsilon \Phi^{-1}(1 - T) + \psi(v - t^*)}{\sigma_2}$. It will also be useful to define

$$G(x, v - t^*) \equiv (1 - \Phi(f(x, v - t^*)))\gamma x.$$

This derivation yields the following result:

Lemma 2 *A function $\hat{\theta}(v - t^*) : \mathbb{R} \rightarrow \mathbb{R}$ is an equilibrium of the game characterizing the revolution stage with positive participation if and only if, for every $v - t^*$ such that $\max_x G(x, v - t^*) \geq k$, it satisfies*

$$G(\hat{\theta}(v - t^*), v - t^*) = k.$$

An equilibrium with a positive probability of successful revolution only exists if it is possible for the condition in Lemma 2 to be satisfied. Since the model is only interesting for parameter configurations where this is possible, I maintain the following assumption throughout.

Assumption 1 *Define $x^*(v - t^*) \equiv \arg \max_x G(x, v - t^*)$.³ Then $\max_v G(x^*(v - t^*), v - t^*) \geq k$.*

Assumption 1 says that there exist realizations of the random variables θ and η such that a positive participation equilibrium exists. (Lemma 9 will formalize exactly what realizations suffice.)

Given that an equilibrium cutoff rule is feasible, it is possible to study some characteristics of such a rule. First, notice that the cutoff rule must be positive, since no one with a negative value of successful regime change would ever participate.

Lemma 3 *Any $\hat{\theta}$ satisfying $G(\hat{\theta}, v - t^*) = k$ is strictly positive.*

Given a realization of $v - t^*$, the function G represents the payoff to a person of type x if x is adopted as the cutoff rule by all members of the population. Understanding the behavior of this function is the key step in characterizing equilibrium in the revolution stage.

Lemma 4 *For all parameter values, $G(x, v - t^*)$ has the following properties:*

1. For $x > 0$, $G(x, v - t^*)$ is increasing in x if and only if $\frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} \geq \alpha$.

³Lemma 4 guarantees that x^* is unique.

2. For $x > 0$, $G(x, v - t^*)$ is single peaked in x .
3. $\lim_{x \rightarrow \infty} G(x, v - t^*) = 0$.

Lemma 4 establishes that for all parameter values, $G(x, v - t^*)$ is non-monotonic and has a single peak in x for positive values of x . The point where, for a realization of v and a belief t^* , it reaches this peak is $x^*(v - t^*)$ as defined in Assumption 1.

The following fact will be useful.

Lemma 5 $G(x^*(v - t^*), v - t^*)$ is increasing in $v - t^*$.

Jointly these Lemmata characterize equilibrium in the revolution stage.

Proposition 1 1. If $G(x^*(v - t^*), v - t^*) > k$, then the only equilibrium of the game characterizing the revolution stage in cutoff strategies is one in which no one participates.

2. If $G(x^*(v - t^*), v - t^*) = k$, then there are two equilibria of the game characterizing the revolution stage in cutoff strategies: one in which no one participates and the other in which players participate if and only if $\theta_i \geq x^*(v - t^*)$.

3. If $G(x^*(v - t^*), v - t^*) < k$, then there are three equilibria of the game characterizing the revolution stage in cutoff strategies: one in which no one participates and two others in which the cutoff rule is given by Equation 3.

The three possibilities discussed in the proposition are illustrated in Figure 1. The first panel shows the situation where the only equilibrium in cutoff strategies involves no participation. The second panel shows the knife edge case where, in addition to an equilibrium with no participation, there is a single equilibrium in cutoff strategies with positive participation. The third panel shows the case where, in addition to the equilibrium with no participation, there are two equilibria in cutoff strategies with positive participation.

In this third case, the intuitions for the two positive participation equilibria are as follows. Consider the effect on the function G of increasing the stringency of the cutoff rule (i.e., requiring a higher signal for participation). Making the cutoff rule more demanding has two competing effects on the expected payoffs from participation to a person whose signal is just at the cutoff. On the one hand, as the signal required becomes more strict, for fixed realizations of the random variables, expected participation decreases and so the probability of victory is lower. This effect, which I call the *probability of winning effect*, tends to make participation less attractive to a person of type $\hat{\theta}$. On the other hand, as the signal required becomes more strict, the signal received by the person whose signal is just at the cutoff rule is better, so this person thinks victory is more likely and thinks the payoff from victory is higher. This effect, which I call the *marginal participant effect*, tends to make participation more attractive to a person of type $\hat{\theta}$.

The presence of two competing effects results in the function G being non-monotonic, which is what leads to multiple positive participation equilibria. In one of these equilibria, all players use a

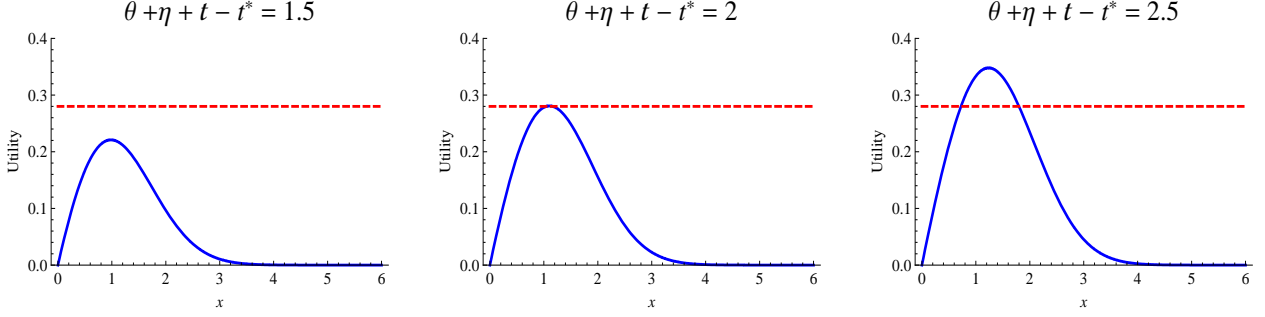


Figure 1: The solid curve represents $G(x, v - t^*)$ —the payoff to a player of type x , should x be adopted as the cutoff. The dashed line represents the costs of participation. Intersections, where $G(x, v - t^*) = k$, are finite cutoff equilibria. The first panel is the case with no finite (i.e., positive participation) cutoff equilibria; the only equilibrium is one in which no one participates. The second panel shows the knife edge case where, in addition to an equilibrium with no participation, there is a cutoff equilibrium with positive participation. The third panel shows the case where, in addition to the equilibrium with no participation, there are two cutoff equilibria with positive participation. For all three panels, parameter values are $m = 2, T = 0.3, \sigma_\epsilon^2 = \sigma_\theta^2 = \sigma_\eta^2 = 1$.

low threshold (i.e., they begin participating at a relatively low level of anti-government sentiment), therefore they believe the probability of a successful revolution is high, therefore they are willing to use the low threshold. In the other, players use a high threshold (i.e., they only participate if they are quite anti-government), therefore they believe the probability of a successful revolution is low, therefore they want to use the high threshold.

When there are two finite cutoff rules consistent with equilibrium, I label them $\hat{\theta}_L(v - t^*)$ and $\hat{\theta}_H(v - t^*)$ for the lower and higher cutoff values, respectively.

Definition 1 *If $G(x^*(v - t^*), v - t^*) > k$ then*

- $\hat{\theta}_L(v - t^*) = \inf \{x \in \mathbb{R}^+ : G(x, v - t^*) = k\}$
- $\hat{\theta}_H(v - t^*) = \sup \{x \in \mathbb{R}^+ : G(x, v - t^*) = k\}$

Let G_1 denote the first derivative of the function G with respect to its first argument. Then the single-peakedness of G implies the following:

Lemma 6 $G_1(\hat{\theta}_L(v - t^*), v - t^*) > 0$ and $G_1(\hat{\theta}_H(v - t^*), v - t^*) < 0$.

2.3 The Vanguard Stage

By engaging in violence, the revolutionary vanguard attempts to change the beliefs of population members about the level of anti-government sentiment in society, in order to foment revolution. As such, the vanguard is only willing to invest in costly violence insofar as doing so increases the probability of a successful revolution. Clearly, if the population is playing the equilibrium with no

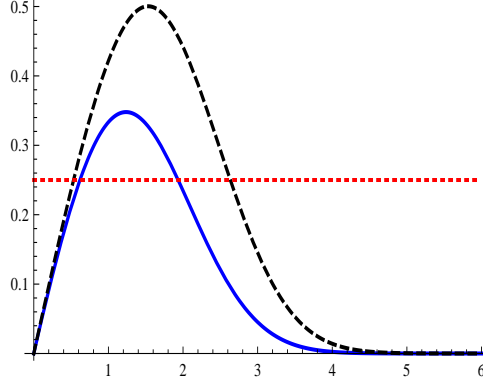


Figure 2: The solid and dashed curves represent $G(x, v - t^*)$ for lower and higher realizations of $v - t^*$, respectively. The dotted line represents the cost of participation. A finite cutoff equilibrium, for a given $v - t^*$, is a point where the line k and the relevant curve intersect. The lower cutoff rule, $\hat{\theta}_L$, is decreasing in $v - t^*$, while the higher cutoff rule, $\hat{\theta}_H$, is increasing in $v - t^*$. Since participation is decreasing in the cutoff rule, effort by the vanguard is counterproductive under the equilibrium selection $\hat{\theta}_H$ but productive under the equilibrium selection $\hat{\theta}_L$.

participation in the revolution stage, then this is not possible, so the vanguard will not engage in violence.

Suppose instead that the population plays an equilibrium with positive participation (i.e., a finite cutoff rule) in the revolution stage. The vanguard wants to exert effort only if doing so increases the probability of a successful revolution. Whether or not this is the case depends on the equilibrium played in the revolution stage.

Consider, first, the case where the population uses the higher cutoff rule, $\hat{\theta}_H$. Suppose vanguard violence convinces the population that the level of anti-government sentiment is higher. They now believe revolution is more likely to succeed, for a given cutoff rule. The result is that some players who were not participating want to participate. This increase in participation means that even lower types are willing to participate, and the equilibrium unravels. To sustain the higher cutoff rule as an equilibrium, when beliefs about the level of anti-government sentiment increases, the cutoff rule has to be *increased*, so that fewer population members will participate. But that means that, from the vanguard's perspective, violence is counterproductive. This argument is illustrated in Figure 2 and formalized in the following lemma.

Lemma 7 *If, when $G(x^*(v - t^*), v - t^*) < k$, the population plays either the equilibrium with no participation or the equilibrium with the higher cutoff rule ($\hat{\theta}_H(v - t^*)$) in the revolution stage, the revolutionary vanguard exerts minimal effort in the vanguard stage ($t = \underline{t}$).*

If the population chooses the lower cutoff rule, $\hat{\theta}_L(v - t^*)$, at the revolution stage, then the level of mobilization is increasing in the realization of v , giving the vanguard incentives to invest in violence. This fact is also illustrated in Figure 2 and formalized in the following lemma.

Lemma 8 For parameter values where it exists, the cutoff rule $\hat{\theta}_L(v - t^*)$ is monotonically decreasing in η , $t - t^*$, and θ .

Suppose the population will play the equilibrium with the lower threshold when it exists. What are the effects of violence by the vanguard on the likelihood of successful revolution?

To answer this question, I need to calculate the *ex ante* probability of a successful revolution. There are two considerations in this calculation. First, a successful revolution is certain not to occur if an equilibrium with positive participation does not exist in the revolution stage. As shown in Proposition 1, a positive participation equilibrium exists if and only if

$$G(x^*(v - t^*), v - t^*) \geq k.$$

Given a realization of η , this condition is satisfied only if the true level of anti-government sentiment is high enough, so that v is sufficiently large.

Lemma 9 There exists a finite $\hat{\theta}(\eta + t - t^*)$ such that a positive participation equilibrium in cutoff strategies exists in the revolution stage if and only if $\theta \geq \hat{\theta}(\eta + t - t^*)$. Moreover $\hat{\theta}(\eta + t - t^*)$ satisfies

$$G(x^*(\hat{\theta} + \eta + t - t^*), \hat{\theta} + \eta + t - t^*) = k \quad (4)$$

and is decreasing in η .

The second consideration is that, as shown in Equation 2, if an equilibrium with a finite cutoff rule is played, then regime change will be successful if and only if

$$\theta \geq \hat{\theta}_L(\theta + \eta + t - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon.$$

The left-hand side of this inequality is obviously increasing in θ and, from Lemma 8, the right-hand side is decreasing in θ . These facts suggest a cutpoint analogous to that identified for an arbitrary cutoff rule in Equation 2, whereby there is victory only if θ is greater than $\theta^*(\eta + t - t^*)$ implicitly defined by

$$\theta^* = \hat{\theta}_L(\theta^* + \eta + t - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon. \quad (5)$$

However, there may not be a θ^* that satisfies this equality. To see why, consider some θ such that

$$\theta > \hat{\theta}_L(\theta + \eta + t - t^*, T) - \Phi^{-1}(1 - T)\sigma_\epsilon.$$

Now begin lowering θ , which lowers the left-hand side and raises the right-hand side of this inequality. Recall that $\hat{\theta}_L(\theta + \eta + t - t^*)$ only exists if $\theta \geq \hat{\theta}(\eta + t - t^*)$. It is possible that prior to lowering θ enough to satisfy the equality in Equation 5, there will cease to be an equilibrium with a finite cutoff rule. If this occurs, then $\hat{\theta}(\eta + t - t^*)$ is the lowest θ where victory could possibly be achieved (since it is the last θ where a positive participation equilibrium exists). And, at $\hat{\theta}(\eta + t - t^*)$

victory will be achieved, since θ had not been lowered enough to achieve equality, which means $\hat{\theta}(\eta + t - t^*) > \hat{\theta}_L(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon$. Finally, notice that at this point, we have that

$$\hat{\theta}_L(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*, T) = x^*(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*).$$

The argument above suggests the following result:

Lemma 10 *For any realization of $\eta + t - t^*$, there is a unique, finite $\bar{\theta}(\eta + t - t^*)$, such that there will be victory if and only if $\theta \geq \bar{\theta}(\eta + t - t^*)$. It is given by*

$$\bar{\theta}(\eta + t - t^*) = \begin{cases} \theta^*(\eta + t - t^*) & \text{if } x^*(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon < \hat{\theta}(\eta + t - t^*) \\ \hat{\theta}(\eta + t - t^*) & \text{else,} \end{cases}$$

where $\theta^*(\eta + t - t^*)$ and $\hat{\theta}(\eta + t - t^*)$ are defined by Equation 5 and Lemma 9, respectively.

Moreover, we have the following two facts:

1. There exists a $\bar{\eta}(t - t^*)$ such that $\bar{\theta}(\eta + t - t^*) = \hat{\theta}(\eta + t - t^*)$ if and only if $\eta \leq \bar{\eta}(t - t^*)$.
2. $\bar{\theta}(\eta + t - t^*)$ is decreasing in $\eta + t - t^*$.

Given the *ex ante* probability of successful regime change, conditional on the level of violence, the vanguard's objective is as follows:

$$\begin{aligned} \max_t \int_{-\infty}^{\bar{\eta}(t-t^*)} \int_{\hat{\theta}(\bar{\eta}+t-t^*)}^{\infty} \frac{1}{\sigma_\theta} \phi\left(\frac{\tilde{\theta} - m}{\sigma_\theta}\right) \frac{1}{\sigma_\eta} \phi\left(\frac{\tilde{\eta}}{\sigma_\eta}\right) d\tilde{\theta} d\tilde{\eta} \\ + \int_{\bar{\eta}(t-t^*)}^{\infty} \int_{\theta^*(\bar{\eta}+t-t^*)}^{\infty} \frac{1}{\sigma_\theta} \phi\left(\frac{\tilde{\theta} - m}{\sigma_\theta}\right) \frac{1}{\sigma_\eta} \phi\left(\frac{\tilde{\eta}}{\sigma_\eta}\right) d\tilde{\theta} d\tilde{\eta} - c(t). \end{aligned} \quad (6)$$

Lemma 11 *In a cutoff equilibrium in which the population uses the cutoff rule $\hat{\theta}_L(\theta + \eta + t - t^*)$, the optimal level of effort by the vanguard, t^* , is characterized by:*

$$\frac{1}{\sigma_\theta \sigma_\eta} \left(\int_{-\infty}^{\bar{\eta}(0)} \phi\left(\frac{\hat{\theta}(\tilde{\eta}) - m}{\sigma_\theta}\right) \phi\left(\frac{\tilde{\eta}}{\sigma_\eta}\right) d\tilde{\eta} - \int_{\bar{\eta}(0)}^{\infty} \phi\left(\frac{\theta^*(\tilde{\eta}) - m}{\sigma_\theta}\right) \frac{\partial \theta^*(\tilde{\eta})}{\partial t} \phi\left(\frac{\tilde{\eta}}{\sigma_\eta}\right) d\tilde{\eta} \right) = c'(t^*). \quad (7)$$

Figure 3 illustrates the vanguard's objective. Victory is achieved if the realization of (η, θ) is to the northeast of the curve defined by $\hat{\theta}$ (the dashed line) and θ^* (the solid curve). Increasing t shifts this curve to the southwest, thereby increasing the probability of successful regime change. The probability is calculated by taking the two-dimension integral above this curve with respect to the distributions of η and θ , as represented in Equation 7.

Increasing the level of violence, relative to expectations, has two effects on the revolutionary vanguard's welfare. First, it increases the probability of a successful revolution. It does this by

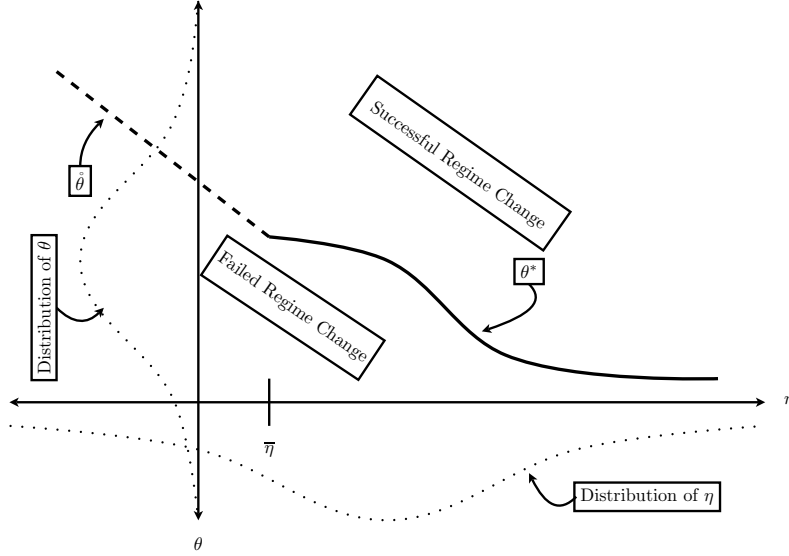


Figure 3: There is successful regime change when the realization of (η, θ) lies to the northeast of the curve defined by $\hat{\theta}$ (the dashed line) and θ^* (the solid curve). Increasing t shifts this curve to the southwest, thereby increasing the probability of successful regime change. The probability is calculated by taking the two-dimension integral of this curve with respect to the distribution of η and θ .

increasing the population's beliefs about the level of anti-government sentiment. When those beliefs are higher, members of the population think that, for any given cutoff rule, revolution is more likely to be successful. At the lower cutoff rule this increased confidence makes more people willing to participate, lowering the equilibrium cutoff rule (and thus decreasing both $\hat{\theta}$ and θ^*), increasing expected mobilization, and increasing the probability of successful revolution. Formally, this can be seen on the left-hand side of Equation 7 where the first term reflects the decrease in $\hat{\theta}$ and the second term reflects the decrease in θ^* . Second, there are costs to resources expended on violence, which can be seen on the right-hand side of Equation 7.⁴

Proposition 2 *Fix parameters and assume Assumption 1 is satisfied. The game has three cutoff equilibria.*

1. *The population chooses never to mobilize and the vanguard chooses minimal effort, \underline{t} .*
2. *The population chooses not to mobilize if $G(x^*(v - t^*), v - t^*) > k$, chooses to mobilize using the higher cutoff rule when $G(x^*(v - t^*), v - t^*) \leq k$, and the vanguard chooses minimal effort, \underline{t} .*
3. *The population chooses not to mobilize if $G(x^*(v - t^*), v - t^*) > k$, chooses to mobilize using the lower cutoff rule when $G(x^*(v - t^*), v - t^*) \leq k$, and the vanguard chooses a level of effort, t^* , given by Equation 7.*

⁴Formally, there is a third effect. The cutpoint in η -space where the binding constraint shifts from $\hat{\theta}$ to θ^* changes with t . However, this turns out not to matter on the margin (see the proof of Lemma 11).

3 Implications for Revolutionary Violence and Regime Change

3.1 Comparative Statics

The model yields several comparative statics for the equilibrium where the population uses the lower cutoff rule. In particular, the level of mobilization, and consequently the probability of successful regime change, is decreasing in the government's capacity to withstand resistance (T) and to impose costs on those who organize against it (k). It is increasing in the extent to which the revolutionary organization excludes those who do not participate from the benefits associated with regime change (γ).

When the probability of success increases (i.e., T or k decrease) or the payoff to success increases (i.e., γ increases) it becomes more attractive to participate. If the threshold for participation remained fixed, then there would be people not participating who could make positive payoffs from participating. At the lower threshold, equilibrium is sustained by making revolution relatively likely to succeed, thereby attracting relatively low types to participate. When revolution becomes more attractive, this equilibrium can accommodate even lower types, so the cutoff rule decreases.

I present these largely intuitive comparative statics primarily because they highlight the fact that structural characteristics of the society can affect the level of mobilization in equilibrium. This point will be important for later discussions.

Proposition 3 *If the equilibrium selection is $\hat{\theta}_L$ when there are multiple equilibria, then the number of people who mobilize, and consequently the probability of successful regime change, is:*

1. *Decreasing in T .*
2. *Decreasing in k .*
3. *Increasing in γ .*

3.2 Sparks, Tinder, and the Problem of Root Causes

So called spark-and-tinder models argue that structural factors—regime capacity, international pressure, grievances, the economy, and so on—can make a society ripe for revolution. These structural factors constitute the tinder. Any spark, the argument goes, can set off the revolutionary fire. Advocates of such models point to the seemingly spontaneous nature of the revolutions in countries such as France and Russia as supporting evidence. (For a classic statement of this view, see Skocpol (1979).)

Yet, as DeNardo (1985), Geddes (1990), and others have pointed out, violent regime changes are rare events. The structural conditions often identified as the root causes of revolution occur far more often than do revolutions themselves. Indeed, this point has been made more generally about the emergence of terrorism and guerilla warfare (i.e., the vanguard). In general, those factors we think of as potential root causes of political violence are far more common than the violence itself. (Krueger (2007), among others, makes this point.)

The model, while not itself a structural account of regime change, casts some doubt on this empirical critique of structuralist explanations. The discussion in the preceding subsection highlighted several parameters of the model that can be interpreted as representing structural features of a society. As shown in Proposition 2, for many values of these parameters the model has multiple equilibria—some where revolution is more likely and some where it is less likely or impossible. The comparative statics in Proposition 3 demonstrate that, given an equilibrium selection, structural factors influence mobilization and the likelihood of a successful revolution. Hence, these structural factors can be viewed as causes of revolution. Nonetheless, if two structurally identical societies play different equilibria, they have very different likelihoods of a successful revolution occurring.

This argument suggests a quite general problem both for the empirical literature on the root causes of political violence and for policymaking. In a world characterized by multiple equilibria, much of the variation in the data may be due to whatever second-order factors determine equilibrium selection, rather than those structural factors that we often think are of first-order importance for explaining political violence. Thus, structural factors may matter (for a given equilibrium selection) but be difficult to detect empirically because we cannot observe which equilibrium a society is playing. Moreover, from the perspective of policymaking, this implies that, even though the data are not well explained by structural variation, it may be that, within a given society (playing its particular equilibrium), changing key structural factors would reduce political violence or the likelihood of violent regime change.

3.3 Vanguard and Selection Effects

Critics of purely structuralist explanations, such as DeNardo (1985), argue that a key problem with such accounts is their failure to consider the importance of revolutionary leaders who attempt to strategically manipulate the masses. The emergence of such leaders, it is argued, is the critical factor that differentiates “structurally ripe” societies that do or do not experience mass political violence.

The model, however, suggests that the fact that the presence of an active revolutionary vanguard is a predictor of a society having a high risk of revolution should not necessarily be interpreted as evidence that vanguards help cause revolutions. In particular, the model predicts that in equilibrium there will be selection effects—even controlling for all relevant structural factors, active vanguards will arise only in those societies that are likely to have successful regime change.

To see why, suppose there are two societies, A and B, that are structurally identical in all relevant respects (i.e., all parameter values are the same). However, in the revolution stage, society A plays the equilibrium with the lower cutoff rule ($\hat{\theta}_L$) and society B plays either the equilibrium with the higher cutoff rule ($\hat{\theta}_H$) or the no participation equilibrium. As shown in Lemma 8 and Proposition 2, an active revolutionary vanguard will emerge in society A but not in B. Moreover, even if the vanguard had no effect on mean beliefs (i.e., $\eta = 0$), society A would be more likely to have a successful revolution than society B because $\hat{\theta}_L < \hat{\theta}_H$. Hence, the fact that the presence of a revolutionary vanguard appears to empirically distinguish societies that do and do not experience

violent regime change (all else equal) may not constitute evidence for the causal importance of vanguards.

3.4 The Efficacy of the Revolutionary Vanguard

The previous subsection points out that any correlation between the presence of a revolutionary vanguard and the probability of a successful revolution could be a pure selection effect. This naturally raises the question: is the vanguard able to increase mobilization and make a successful revolution more likely? A step toward an answer is the following result.

Proposition 4 *The level of mobilization is increasing in the level of violence by the vanguard, v .*

This proposition points to a sense in which violence by a vanguard can help to mobilize a population toward revolution. For fixed beliefs about effort by the vanguard, higher realizations of violence increase mobilization. Thus, the model is consistent with cases where successful vanguards seem to ignite mass uprisings against a government. Indeed, the model predicts that higher-than-expected levels of a violence by a vanguard will increase mobilization and the probability of regime change.

Proposition 4 shows that *ex post* higher levels of violence improve the likelihood of a revolution succeeding. This fact notwithstanding, from an *ex ante* perspective the vanguard is clearly not efficacious, in the following sense. If the vanguard could commit to a level of effort \underline{t} , the level of mobilization and the probability of successful regime change would not be effected. The reason is as follows.

In equilibrium, the population correctly anticipates the level of effort the vanguard exerts. As a result, the population is able to filter out vanguard effort from the level of violence and extract an unbiased signal of the level of anti-government sentiment from the level of violence. That is, on the equilibrium path, $v - t^*$ is equal to $\theta + \eta$, which is a normally distributed random variable with mean equal to the true level of anti-government sentiment θ . As such, while the vanguard attempts to manipulate the population's beliefs, in expectation it does not actually succeed at doing so. The vanguard nonetheless exerts effort because, if it did not do so the population would likely observe lower-than-expected levels of violence and conclude that the level of anti-government sentiment is lower than it is in reality. However, if the vanguard could commit to low effort, the population would then update based on that commitment. As a result, the *ex ante* probability of a successful revolution would be unchanged.⁵

⁵A related question is whether the probability of successful regime change is higher, *ex ante*, with a vanguard than in a modified game where no vanguard is present, so no signal is sent. That is, the vanguard may have an effect simply by changing the precision of the population's beliefs, rather than the mean. Answering this question has proven to be very technically difficult, but preliminary explorations suggest that it can go either way, depending on parameter values.

3.5 Vanguard and the Level of Revolutionary Extremism

The model generates two equilibria with positive probability of successful regime change. In the first, the revolutionary vanguard is quite active and the population uses the lower cutoff strategy ($\hat{\theta}_L$). In the second, the revolutionary vanguard is inactive and the population uses the higher cutoff strategy ($\hat{\theta}_H$).

In the first equilibrium, more people mobilize in expectation, because they are using the lower cutoff rule. As a result, successful revolution is more likely. Further, the average level of anti-government sentiment among those who participate in revolution is lower in the first equilibrium, since the lower cutoff rule induces people with lower signals to participate.

This argument suggests some empirical implications regarding different types of outcomes in revolutions that occur with and without the emergence of an active revolutionary vanguard.⁶ These implications are summarized in the following result.

Proposition 5 *Holding fixed parameter values, revolutions that begin with non-minimal levels of effort by a revolutionary vanguard ($t > \underline{t}$) have:*

1. *higher expected levels of mobilization,*
2. *higher probabilities of success, and*
3. *lower average levels of anti-government sentiment among the participants,*

as compared to revolutions where the vanguard engages in minimal effort.

3.6 Other Mechanisms for Organizing Revolution

I have highlighted one previously unexplored micro-foundation for a credible revolutionary threat. In particular, how a revolutionary vanguard might use violence to coordinate and mobilize a population against a government by communicating information about the level of anti-government sentiment in society. This mechanism is by no means the only one considered in the literature.

Scholars have discussed other pathways through which violence by a revolutionary vanguard can foment revolution. Ginkel and Smith (1999) argue that a revolutionary vanguard that has private information regarding the strength of the government can, under certain conditions, engage in costly violence that will signal to the population that the government is likely to be weak and, therefore, susceptible to revolution. Baliga and Sjoström (2009) study how violence by terrorists (modeled as cheap-talk communication) with private information about the preferences of one party in a dispute can cause a cycle of violence by goading that party into increased aggressiveness. Bueno de Mesquita and Dickson (2007) and Siqueira and Sandler (2007) argue that violence by an extremist group can be used to provoke a government to engage in repression and thereby spark a more widespread violent backlash. And, of course, a variety of scholars have discussed the

⁶I thank Jim Morrow for suggesting this line of argument.

role of revolutionary entrepreneurs in solving the collective action problem by providing selective incentives.

Other papers discuss different roles for information in similar revolutionary settings. Edmond (2007, 2008) explores how regimes may attempt to manipulate information to convince citizens not to participate in revolution. Lohmann (1994) studies how sudden revolution can occur as a result of information cascades.

Another related literature explores how non-violent forms of political activity can provide informational signals that can help coordinate a credible revolutionary threat. Fearon (2006) models a situation in which citizens attempting to create a credible revolutionary threat face a coordination problem (due to the need to impose punishments in a repeated game) and uncertainty about each other's preferences. He argues that regularly timed elections can coordinate the revolutionary threat (if elections are not held, rebel) and simultaneously provide an aggregate signal of the level of public dissatisfaction with the government. Elections in that model and the revolutionary vanguard in this model, thus, play related roles.

4 Global Games of Regime Change and Equilibrium Uniqueness

The model above is quite similar to models from the literature on global games of regime change, which have become workhorse models for studying currency attacks, bank runs, political regime change, and so on (Angeletos, Hellwig and Pavan 2006, 2007; Guimaraes and Morris 2007; Edmond 2007). In static, exogenous information global games of regime change, as long as private signals are sufficiently informative relative to the public prior, there is a unique equilibrium. However, in a static, exogenous information version of my model, regardless of informational assumptions, there are always multiple equilibria. Understanding why may be useful for building intuitions about the forces at work in global games of regime change and the verisimilitude, for applied settings, of the assumptions needed to generate uniqueness.⁷

To build intuition, it is useful to start with a canonical example of a global game (Carlsson and van Damme 1993; Morris and Shin 1998). This particular example is most closely related to the motivating example in Morris and Shin (2005). The game builds on the following two-player game of complete information (the cells only contain Player 1's payoffs, the game is symmetric).

		Player 2	
		$a_2 = 0$	$a_2 = 1$
Player 1	$a_1 = 0$	0	0
	$a_1 = 1$	$\theta - k$	θ

⁷I am particularly indebted to Amanda Friedenberg for helping me think through these issues. However, I alone bare responsibility for any wrong-headed conclusions.

The idea, here, is that a player who participates ($a_i = 1$) makes profit θ , but bears cost k if she participates alone.⁸

Uncertainty is introduced as follows. Assume k is fixed and positive, but the common payoff parameter θ is distributed normally on the real line with mean m and variance σ_θ^2 . Each player receives as signal $s_i = \theta + \epsilon_i$, where the ϵ 's are iid draws from a mean zero normal distribution with variance σ_ϵ^2 .

Suppose we look for cutoff equilibria of the form, choose $a_i = 1$ if and only if $s_i \geq \hat{s}$. After observing s_i , a player's posterior is that θ is distributed normally with mean $\lambda s_i + (1 - \lambda)m$ and variance $\sigma^2 = \lambda \sigma_\epsilon^2$, with $\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}$. His subjective assessment of the probability that $s_{-i} < \hat{s}$ is $\Phi\left(\frac{\hat{s} - \lambda s_i - (1 - \lambda)m}{\sqrt{\sigma}}\right)$. Thus, Player 1 chooses $a_1 = 1$ if and only if

$$\lambda s_1 + (1 - \lambda)m - \Phi\left(\frac{\hat{s} - \lambda s_i - (1 - \lambda)m}{\sqrt{\sigma}}\right) k \geq 0.$$

As Morris and Shin (2005) point out, the term $\Phi\left(\frac{\hat{s} - \lambda s_i - (1 - \lambda)m}{\sqrt{\sigma}}\right)$ represents player 1's strategic uncertainty about what player 2 will do, conditional on player 1's signal.

A cutoff rule must satisfy

$$\lambda \hat{s} + (1 - \lambda)m - \Phi\left(\frac{(\hat{s} - m)(1 - \lambda)}{\sqrt{\lambda \sigma \epsilon}}\right) k = 0. \quad (8)$$

Again, the term $\Phi\left(\frac{(\hat{s} - m)(1 - \lambda)}{\sqrt{\lambda \sigma \epsilon}}\right)$ is player 1's strategic uncertainty about what player 2 will do, now conditional on player 1's signal having been just at the cutoff rule \hat{s} .

Suppose $\sigma_\epsilon \rightarrow 0$ or $\sigma_\theta^2 \rightarrow \infty$. Then player 1's conditional probability of player 2's signal being beneath the cutoff rule becomes insensitive to player 1's signal. This is because either player 1's signal tells him with virtual certainty what player 2's signal is ($\sigma_\epsilon \rightarrow 0$) or because the prior is so diffuse that, no matter his signal, player 1 does not revise his beliefs ($\sigma_\theta^2 \rightarrow \infty$). In either case, Equation 8 has a unique solution. In particular, taking these limits yields a unique $\hat{s} = k/2$.

More generally, Morris and Shin (2005) show that, as long as the signal variance is sufficiently small relative to the prior variance in this game, there will be a unique equilibrium. They give the following intuition for their theorem showing joint sufficiency of several conditions for uniqueness: "uniqueness follows from the insensitivity of strategic uncertainty with respect to shifts in a player's own type." (p. 219)

Now consider the standard payoff structure for a global game of regime change (Angeletos, Hellwig and Pavan 2006, 2007; Guimaraes and Morris 2007; Edmond 2007). There is a continuum of individuals (of measure 1), each of whom makes a binary choice, $a_i \in \{0, 1\}$. There is regime change if the measure of people choosing $a_i = 1$, labeled N , is greater than a threshold T . Choosing to participate imposes cost k on the participant. Regime change yields a payoff of θ to the participant. Payoffs for a representative player are given by the following matrix.

⁸For applications to questions of political violence that build on related models, see Baliga and Sjoström (2004); Chassang and Padro i Miguel (2006, 2007).

		$N < T$	$N \geq T$
Player i	$a = 0$	0	0
	$a = 1$	- k	$\theta - k$

Payoffs for a Representative Player in a Regime Change Game

Notice that in addition to describing payoffs in the complete information games underlying standard global games of regime change, this payoff matrix also describes the payoffs in the complete information game underlying the revolution stage of my game, assuming $\gamma = 1$ and that there is no heterogeneity in payoffs.

Now, suppose we introduce the same sort of payoff uncertainty to this game that was introduced by Carlsson and van Damme (1993) and Morris and Shin (1998) in standard global games (and in the example above). That is, let θ be a normally distributed random variable with mean m and variance σ_θ^2 and let each player receive a signal $s_i = \theta + \epsilon_i$, where the ϵ 's are as above. Call this game Γ^θ (notice, this is the same sort of uncertainty as in my model).

This incomplete information game has the payoff structure of a global game of regime change and the same sort of uncertainty as in standard global games. It would seem that the intuition above—that uniqueness obtains as long as strategic uncertainty is sufficiently insensitive to signals—should hold. However, it does not. This game fails to generate equilibrium uniqueness in finite cutoff strategies in exactly the same manner as the revolution stage of my game above.

Proposition 6 *In the game Γ^θ , if there is a positive participation equilibrium, then there are (almost always) two positive participation equilibria. Moreover, a third equilibrium, with no participation, always exists.*⁹

The intuition for multiplicity here is exactly as above. There are two substantive effects—the “probability of winning” and “marginal participant” effects—which pull in opposite directions.

How do extant global games of regime change avoid this multiplicity result? They change how uncertainty enters into the model. In the original global games models, and in Γ^θ , uncertainty was over the payoff from achieving the coordinated outcome. In global games of regime change, uncertainty is over the threshold needed to achieve the coordinated outcome. That is, global games of regime change assume that θ is fixed and known, but there is uncertainty over the threshold T , which is normally distributed on the real line. Players receive signals $t_i = T + \xi_i$, where $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$. Label the game where players receive these signals and face the representative payoff matrix above as Γ^T . I begin by establishing that the standard uniqueness result holds.

⁹Appendix B shows that all such equilibria are stable with respect to standard static notions of equilibrium stability.

Proposition 7 (*Angeletos, Hellwig and Pavan (2007)*) *If $\sigma_\xi < \sigma_T^2 \sqrt{2\pi}$, then the game Γ^T has a unique equilibrium in cutoff strategies.*

Given these results, three questions arise: First, why does the regime change game not generate uniqueness with the standard type of uncertainty introduced in earlier global games. Second, why does relocating the uncertainty away from the payoffs and to the threshold return a standard-looking global games uniqueness result? Third, what does this imply, substantively, about uniqueness results in regime change games?

4.1 The Underlying Complete Information Games

One way to start thinking about why the two models yield such different predictions is to focus on the difference between the complete information games underlying Γ^θ and Γ^T .

Suppose, in the game Γ^θ , that the true state θ is known. If $\theta < \frac{k}{\gamma}$, then there is a unique equilibrium with no participation. If $\theta > \frac{k}{\gamma}$, then there are three equilibria. The first has zero participation, the second has full participation, and the third is in mixed strategies.

Importantly, there are sufficiently low realizations of θ such that non-participation is a dominant strategy. However, there are no sufficiently high realizations of θ such that participation is a dominant strategy. That is, Γ^θ does not satisfy the assumption of “Limit Dominance”, which is one of the standard jointly sufficient conditions employed in uniqueness proofs for global games.

Such is not the case for the game Γ^T . If the realization of T is negative, then participation is a dominant strategy, since victory is certain even if only one measure zero person participates. If the realization of T is greater than 1, then not participating is a dominant strategy, since victory is impossible even if everyone participates. Thus, Γ^T satisfies limit dominance.

This difference between the two underlying complete information games is key to understanding the difference between the equilibrium correspondences of the incomplete information games Γ^θ and Γ^T . As we have already seen, the standard intuition about insensitive strategic uncertainty driving uniqueness is incomplete, as it does not explain the difference between Γ^θ and Γ^T . Below, I show that it is the combination of limit dominance and insensitivity of strategic uncertainty that drives uniqueness results in global games of regime change. To understand how, it is useful to study the equilibria in the games Γ^θ and Γ^T in more detail.

4.2 A Substantive Intuition for Uniqueness and Multiplicity

I start by deriving the key equilibrium conditions in Γ^θ and Γ^T .

In the game Γ^θ , if cutoff rule \hat{s} is adopted, then there is victory if $\theta \geq \theta^*(\hat{s})$ given by

$$1 - \Phi\left(\frac{\hat{s} - \theta^*}{\sigma_\epsilon}\right) = T. \tag{9}$$

This implies that if x is adopted as the cutoff rule, then a person who received signal x has a payoff

from participating given by:

$$G^\theta(x) = \left(1 - \Phi\left(\frac{\theta^*(x) - \lambda x - (1 - \lambda)m}{\sigma_\epsilon \sqrt{\lambda}}\right)\right) \gamma(\lambda x + (1 - \lambda)m).$$

Thus, a cutoff rule \hat{s} is an equilibrium if and only if:

$$G^\theta(\hat{s}) = k.$$

In the game Γ^T , if a cutoff rule \hat{t} is adopted, then there is victory if $T \leq T^*(\hat{t})$ given by

$$\Phi\left(\frac{\hat{t} - T^*}{\sigma_\xi}\right) = T^*. \quad (10)$$

A person who observes signal t_i has posteriors that are normally distributed with mean $\bar{m}_i = \beta t_i + (1 - \beta)m$ and variance $\beta\sigma_\xi^2$, with $\beta = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_\xi^2}$. If x is adopted as the cutoff, then the expected payoff of participating for a person who received signal x is given by:

$$G^T(x) = \Phi\left(\frac{T^*(x) - \beta x - (1 - \beta)m}{\sigma_\xi \sqrt{\beta}}\right) \gamma\theta.$$

A cutoff rule \hat{t} is an equilibrium if and only if:

$$G^T(\hat{t}) = k.$$

The function G^θ has all the same properties as the function G in the main analysis, yielding multiplicity. Let's focus on the game Γ^T . Differentiating the associated function, G^T , with respect to the cutoff rule yields:

$$\frac{\partial G^T}{\partial x} = \phi\left(\frac{T^*(x) - \beta x - (1 - \beta)m}{\sigma_\xi \sqrt{\beta}}\right) \frac{\gamma\theta}{\sigma_\xi \sqrt{\beta}} \left(\frac{\partial T^*}{\partial x} - \beta\right). \quad (11)$$

Consider the effect of increasing the stringency of the cutoff rule in Γ^T (i.e., decreasing x). Just as in the analysis of the main game in Section 2, making the cutoff rule more demanding has two competing effects on the expected payoffs from participation to a person whose signal is just at the cutoff rule (i.e., on the function G^T). These two effects are captured by the term $(\frac{\partial T^*}{\partial x} - \beta)$ in Equation 11. On the one hand, as the signal required becomes more stringent, for a fixed realization of T , expected participation decreases and so the probability of victory is lower. This *probability of winning effect*, which is captured by the term $\frac{\partial T^*}{\partial x}$, tends to make the function G^T decreasing in the stringency of the cutoff rule (i.e., make $\frac{\partial G^T}{\partial x}$ positive). On the other hand, as the signal required becomes more stringent, the signal received by the person whose signal is just at the cutoff rule is better, so this person thinks victory is more likely, making participation more attractive. This *marginal participant effect*, which is captured by the term β , tends to make the function G^T increasing in the stringency of the cutoff rule (i.e., make $\frac{\partial G^T}{\partial x}$ negative).

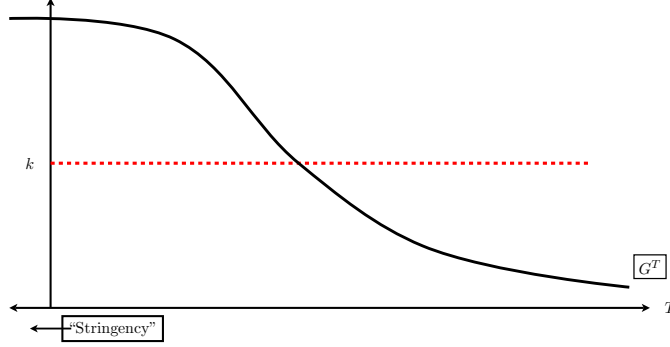


Figure 4: G^T is decreasing (i.e., increasing in stringency) when $\sigma_\xi < \sigma_T^2 \sqrt{2\pi}$. Under this assumption, the game Γ^T has a unique equilibrium.

The standard informational assumption that yields uniqueness (i.e., $\sigma_\xi < \sigma_T^2 \sqrt{2\pi}$) insures that β is larger than $\frac{\partial T^*}{\partial x}$. This situation is illustrated in Figure 4.

Substantively, why does this informational assumption yield equilibrium uniqueness? It is clear that the one point where G^T crosses k is an equilibrium. So the key question is why there are no other equilibria.

First, consider the possibility of an equilibrium with very low or zero participation (i.e., participate if and only if t_i less than some very small \hat{t}). Notice that for very low values on the x -axis, the function G^T lies well above k . This reflects the fact that a person who receives a very good signal (i.e., t_i very small) wants to participate, even if she believes that essentially no one else will participate. Why is this the case?

The answer comes from a combination of limit dominance and the informational assumption. Limit dominance says that there exists a “lower dominance region” of values of T (here $T < 0$). When T lies in this lower dominance region a person should participate, *even if no one else will participate*, because that person’s participation alone would topple the regime. The informational assumptions require that (i) the prior distribution has thick tails (σ_T^2 large) so that it is likely *ex ante* that T lies in the lower dominance region and (ii) signals are precise (σ_ξ small). Taken together, these assumptions insure that a person who received a very good (i.e., low) signal is virtually certain that T lies in the lower dominance region and, consequently, that she alone is sufficient for regime change. Thus, she wants to enter even if no one else will and so there are no equilibria with zero, or very low, participation.

A similar argument holds for the case of full participation (i.e., $\hat{t} \rightarrow \infty$). Given thick tails and precise signals, a person who receives a high signal is virtually certain that T is so large that regime change is impossible, *even if everyone participates*. Thus, such a person does not want to participate even if everyone else is participating. Hence, there is no full participation equilibrium.

The arguments above provide an account of why the game Γ^T has only one equilibrium if $\sigma_\xi > \sigma_T^2 \sqrt{2\pi}$. However, if this information assumption does not hold, then Γ^T generates multiple equilibria. This situation, where the probability of winning effect dominates the marginal

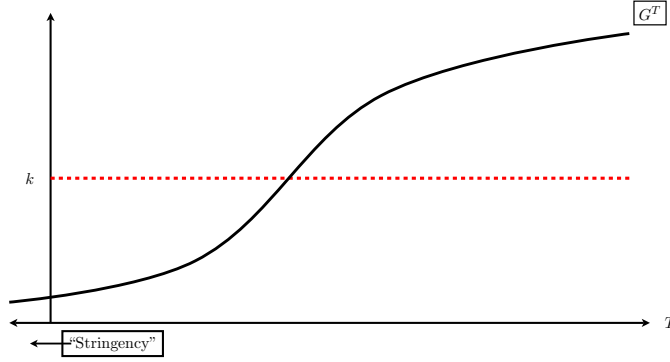


Figure 5: The game Γ^T with multiple equilibria.

participant effect, is illustrated in Figure 5.

Again, it is clear why the point where $G^T = k$ is an equilibrium. However, now there are two other equilibria: one with zero participation and one with full participation.

Consider the case of zero participation (i.e., $\hat{t} \rightarrow \infty$). Since the tails are thinner, relative to the precision of private information, any person believes it is very unlikely that she alone could topple the regime, regardless of how good her signal is. Thus, a person who believes no one will participate does not want to participate, even if she receives a very good signal, yielding an equilibrium with no participation.

The case of full participation is similar. Given thin tails and precise signals, even a person who receives a very bad signal finds it unlikely that the regime cannot be toppled by full participation. Thus, if everyone is expected to participate, everyone wants to participate, even people whose signals were very high.

As the main analysis already highlights, the game Γ^θ always has multiple equilibria. This is because, as already discussed, that game has no upper dominance region (i.e., it is never a dominant strategy to participate). As such, no matter the informational assumptions, a person never wants to participate if no one else intends to, so the argument used in the game Γ^T to rule out equilibria when signals are very accurate and tails are thick does not hold in Γ^θ .

4.3 Implications

The discussion above provides a fuller intuition for uniqueness results in global games of regime change. Such results hinge on two key assumptions. First, the underlying complete information game satisfies limit dominance—i.e., there are parameter values such that participating and not participating are dominant strategies. Second, the informational assumptions (thick tails and precise signals) insure $\sigma_\xi < \sigma_T^2 \sqrt{2\pi}$ that, conditional on receiving a signal in the tail, a person believes it is very likely that the game is in one of its dominance regions. That is, a person who receives a particularly good signal must be virtually certain that she alone could topple the regime and a person who receives a particularly bad signal must be virtually certain that the regime would not fall even if everyone participated. Taken together, these assumptions yield uniqueness.

Put this way, standard uniqueness results seem somewhat less than satisfying for applications. In particular, for applications such as bank runs, currency crises, or political regime change, it is hard to imagine that any player believes that she alone has any chance of toppling the regime without the participation of others.¹⁰ Yet, without such a possibility having strictly positive support (and a positive measure of people who believe it), regime change games are characterized by multiplicity, even with uncertainty over the threshold. Moreover, under more standard global games assumptions about the locus of uncertainty (i.e., uncertainty over payoffs), there is multiplicity regardless of informational assumptions.

Given these two facts, it seems reasonable to argue that, substantively speaking, regime change games may be more naturally characterized by equilibrium multiplicity. And the arguments in this paper have attempted to suggest that such multiplicity may actually be important for understanding the strategy of revolution making and the empirical data.

5 Conclusion

I study how a vanguard may use violence to coordinate and mobilize members of a mass public by convincing them that the level of anti-government sentiment in society is high. The model is consistent with the idea that violence by vanguards can sometimes spark successful revolution. Higher than expected realizations of violence lead to increased mobilization. However, the model also suggests that the micro-foundations of revolution in general, and the role of vanguards in particular, are complicated and subtle.

The model has multiple equilibria, some where successful regime change is very likely and some where it is very unlikely or even impossible. Within an equilibrium, structural factors affect the likelihood of revolution. Nonetheless, if two identical societies play different equilibria, they have very different likelihoods of experiencing revolution. Within an equilibrium, comparative static analysis shows that structural factors affect the likelihood of revolution. Nonetheless, if two structurally identical societies play different equilibria, they have very different likelihoods of experiencing revolution. This finding implies that it may be very difficult to empirically identify root causes of political violence. Moreover, it suggests that the standard empirical critique of structural accounts—that many more societies possess the putative structural causes of revolution than actually experience revolution—may have weaker logical foundations than the current literature acknowledges.

The characterization of the multiple equilibria also establishes the presence of selection effects. Active revolutionary vanguards only emerge in societies that are already playing the equilibrium where revolution is likely. Thus, even if vanguard violence is ineffective, a society with an active vanguard will be more likely to have a revolution (all else equal) than a society without a vanguard. These selection effects further complicate attempts to establish, empirically, the causes of revolution.

¹⁰One possible interpretation that could partially get around this point is to think about individuals playing correlated strategies, so that “players” in the game actually represent larger groups of agents. Of course, one has to find a way to reconcile this interpretation with the fact that, within the model, all players are measure zero.

In addition to these substantive implications, I argued that the multiplicity of equilibrium helped shed some light on the source and verisimilitude of uniqueness results in the applied literature on global games of regime change.

Appendix A: Proofs of Numbered Results

Proof of Lemma 1. Suppose players play a strategy profile with $a_i = 0$ for all θ_i . The probability of victory is 0. If an individual were to consider deviating to participation, the probability of victory would still be zero, since all individuals are measure 0. Thus, the payoff to the deviation is $-c$, while the payoff to not participating is $0 > -k$. ■

Proof of Lemma 2. Necessity follows from the argument in the text. This implies that if $\max_x G(x, v - t^*) < k$, then there can be no equilibrium with a finite cutoff rule.

For sufficiency, consider a profile in the revolution stage where all players employ such a cutoff rule. Fix a $v - t^*$ and consider a player with type $\theta_i < \hat{\theta}(v - t^*)$. Such a player will participate if $[1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\theta_i - k \geq 0$. Now, notice that

$$\begin{aligned} [1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\theta_i - k &< [1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\hat{\theta}(v - t^*) - k \\ &= G(\hat{\theta}(v - t^*), v - t^*) - k \\ &= 0. \end{aligned}$$

Thus, there is no profitable deviation to participating. Now consider a person with type $\theta_i > \hat{\theta}(v - t^*)$. Such a person will participate if $[1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\theta_i - k \geq 0$. Now, notice that

$$\begin{aligned} [1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\theta_i - k &> [1 - \Phi(f(\hat{\theta}(v - t^*), v - t^*))]\gamma\hat{\theta}(v - t^*) - k \\ &= G(\hat{\theta}(v - t^*), v - t^*) - k \\ &= 0. \end{aligned}$$

Thus, there is no profitable deviation to not participating.

By construction, a person of type $\theta_i = \hat{\theta}(v - t^*)$ is indifferent. ■

Proof of Lemma 3. Since $k > 0$, at any $\hat{\theta}$ that satisfies Equation 3, $G(\hat{\theta}, v - t^*)$ must be positive. Since $1 - \Phi(f(x, v - t^*)) > 0$ for all x , this means that for $G(\hat{\theta}, v - t^*) = (1 - f(\hat{\theta}, v - t^*))\hat{\theta}$ to be positive, $\hat{\theta}$ must be positive. ■

Proof of Lemma 4.

The first point follows directly from differentiating and rearranging.

Next I prove the second point. From the first point, $G(x, v - t^*)$ is increasing in x if and only if $\frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))}$ is greater than the finite positive constant α . Further, $f(x, v - t^*)$ is clearly increasing in x . Now, notice that since the normal density is log-concave, $\frac{1 - \Phi(f(x, v - t^*))}{\phi(f(x, v - t^*))}$ is decreasing monotonically in x , which implies that $\frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))}$ is decreasing monotonically in x , for $x > 0$. Thus, to prove that $G(x, v - t^*) = (1 - \Phi(f(x, v - t^*)))x$ is single peaked in x for $x > 0$, it is sufficient to show that there exists an sufficiently small that $\frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} > \alpha$ and an x sufficiently large that $\frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} < \alpha$. If this is true, the fact that $G(x, v - t^*)$ is continuous and its slope is

monotonically decreasing will imply single peakedness.

I start by showing that $\lim_{x \rightarrow 0} \frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} = \infty$. To see this, note that the limit of the numerator as x goes to 0 is some positive finite number and the limit of the denominator is zero. Thus, for $x > 0$ sufficiently small, $G(x, v - t^*)$ is increasing.

Next I show that there is a sufficiently large x that $G(x, v - t^*)$ is decreasing. To see this, first note that $G(1, v - t^*)$ is strictly positive. Next the following chain of inequalities shows that $\lim_{x \rightarrow \infty} \frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} = 0$:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{1 - \Phi(f(x, v - t^*))}{x\phi(f(x, v - t^*))} &= \lim_{x \rightarrow \infty} \frac{-\phi(f(x, v - t^*))f_1(x, v - t^*)}{\phi(f(x, v - t^*)) + x\phi'(f(x, v - t^*))f_1(x, v - t^*)} \\ &= \lim_{x \rightarrow \infty} \frac{-\phi(f(x, v - t^*))f_1(x, v - t^*)}{\phi(f(x, v - t^*)) - xf(x, v - t^*)\phi(f(x, v - t^*))f_1(x, v - t^*)} \\ &= \lim_{x \rightarrow \infty} \frac{f_1(x, v - t^*)}{xf(x, v - t^*)f_1(x, v - t^*) - 1} \\ &= 0, \end{aligned}$$

where $f_1(x, v - t^*) = \alpha$ is the partial derivative of f with respect to its first argument (x). The first equality is due to l'Hopital's rule, the second equality uses the fact that $\phi'(x) = -x\phi(x)$, the third equality is algebra, and the fourth equality follows from the fact that $f(x, v - t^*)$ is increasing in x and $f_1(x, v - t^*) = \alpha$ is constant in x . These equalities show that at least somewhere between $x = 1$ and the limit as x goes to infinity, $G(x, v - t^*)$ is decreasing. And the fact that the derivative of $G(x, v - t^*)$ is monotonically decreasing for positive x implies that whenever $(G(x, v - t^*))$ first slopes down, it slopes down forever after, establishing that there is a single peak for positive x .

Finally I prove the third enumerated point. We can write $f(x, v - t^*) = \alpha x - \beta$. Now we can

state the following chain of equalities:

$$\begin{aligned}
\lim_{x \rightarrow \infty} (1 - \Phi(f(x, v - t^*)))x &= \lim_{x \rightarrow \infty} \frac{(1 - \Phi(f(x, v - t^*)))}{\frac{1}{x}} \\
&= \lim_{x \rightarrow \infty} \phi(f(x, v - t^*))f_1(x, v - t^*)x^2 \\
&= \lim_{x \rightarrow \infty} \frac{f_1(x, v - t^*)x^2 e^{-\frac{f(x, v - t^*)^2}{2}}}{\sqrt{2\pi}} \\
&= \lim_{x \rightarrow \infty} \frac{f_1(x, v - t^*)x^2}{e^{\frac{f(x, v - t^*)^2}{2}}\sqrt{2\pi}} \\
&= \lim_{x \rightarrow \infty} \frac{f_1(x, v - t^*)2x}{f(x, v - t^*)f_1(x, v - t^*)e^{\frac{f(x, v - t^*)^2}{2}}\sqrt{2\pi}} \\
&= \lim_{x \rightarrow \infty} \frac{\alpha 2x}{(\alpha x - \beta)\alpha e^{\frac{(\alpha x - \beta)^2}{2}}\sqrt{2\pi}} \\
&= \lim_{x \rightarrow \infty} \frac{\alpha 2}{\alpha^2 e^{\frac{(\alpha x - \beta)^2}{2}}\sqrt{2\pi} + (\alpha x - \beta)^2 \alpha^2 e^{\frac{(\alpha x - \beta)^2}{2}}\sqrt{2\pi}} \\
&= 0,
\end{aligned}$$

where, in order, the equalities follow from (1) simple rearrangement, (2) l'Hospital's rule, (3) the definition of the PDF of the standard normal, (4) the fact that $e^{-x} = 1/e^x$, (5) l'Hospital's rule, (6) the definition $f(x, v - t^*) = \alpha x - \beta$, (7) l'Hospital's rule, and (8) the observation that the numerator is a positive constant in x and the denominator goes to infinity. ■

Proof of Lemma 5. I make use of the following Claim.

Claim 1 For any $x > 0$, $G(x, v - t^*)$ is increasing in $v - t^*$.

The value of x^* is clearly always positive, since $1 - \Phi(f(x, v - t^*))$ is strictly positive.

Now, given the claim, fix an $x^*(v - t^*)$. Now consider some new $\overline{v - t^*} > v - t^*$. It is clear from the claim that $G(x^*(v - t^*), \overline{v - t^*}) > G(x^*(v - t^*), v - t^*)$. Thus, by the definition of a maximum, it must be that $G(x^*(\overline{v - t^*}), \overline{v - t^*}) > G(x^*(v - t^*), v - t^*)$.

Now all the remains is to prove the claim.

Proof of Claim. We can write

$$G(x, v - t^*) = \left[1 - \Phi \left(\frac{(1 - (1 - \psi)\lambda)}{\sigma_2} x - \frac{(1 - \psi)(1 - \lambda)m + \sigma_\epsilon \Phi^{-1}(1 - T) + \psi(v - t^*)}{\sigma_2} \right) \right] \gamma x$$

Differentiating, we have

$$\frac{\partial G(x, v - t^*)}{\partial (v - t^*)} = \phi(f(x, v - t^*)) \frac{\psi \gamma x}{\sigma_2} > 0.$$

■ ■

Proof of Proposition 1. From Lemma 1, there is always an equilibrium with no participation.

Now, from Lemma 3, we know that any equilibrium cutoff rule has a positive threshold and from Lemma 2 it must satisfy Condition 3. Thus, to find equilibria in cutoff strategies it suffices to look at positive values of x such that $G(x, v - t^*) = k$.

To see that the only equilibrium in cutoff strategies when $G(x^*(v - t^*), v - t^*) < k$ is the one with no participation, notice that when this holds, the condition in equation 3 does not hold for any θ_i . Thus there cannot be another equilibrium in cutoff strategies.

To see that there is one other equilibrium in cutoff strategies when $G(x^*(v - t^*), v - t^*) = k$ first recall from Lemma 4 that $G(x, v - t^*)$ has a single peak and that at that peak, it is equal to $G(x^*(v - t^*), v - t^*)$. Since $G(x^*(v - t^*), v - t^*) = k$, $x^*(v - t^*)$ is an equilibrium in cutoff strategies by Lemma 2. Moreover, $G(x, v - t^*)$ is less than k for all $x \neq x^*(v - t^*)$, by the definition of $x^*(v - t^*)$, so there are no other equilibria in cutoff strategies, again by Lemma 2.

Finally when $G(x^*(v - t^*), v - t^*) > k$ there are two equilibria in cutoff strategies with positive participation. To see this, notice that $G(0, v - t^*) = 0$. Moreover, from the proof of Lemma 4, the limit of $G(x, v - t^*)$ as x goes to infinity is also 0. Since the LHS is continuous and single peaked with peak at $G(x^*(v - t^*), v - t^*)$, it follows that, for $x \in (0, \infty)$, $G(x, v - t^*)$ takes all values in $(0, G(x^*(v - t^*), v - t^*))$ twice. Since $G(x^*(v - t^*), v - t^*) > k$, this implies that $G(x, v - t^*)$ takes the value k twice for $x \in (0, \infty)$, each instance of which is an equilibrium in cutoff strategies. ■

Proof of Lemma 6. By Lemma 4 $G(x, v - t^*)$ is single peaked in its first argument. The combination of single peakedness and the fact that $G(\hat{\theta}, v - t^*) = k$ at all equilibria with finite $\hat{\theta}$, implies that $\hat{\theta}_L(v - t^*)$ and $\hat{\theta}_H(v - t^*)$ are on opposite sides of $x^*(v - t^*)$. Since $x^*(v - t^*)$ is the maximum and G is single peaked, to the left of $x^*(v - t^*)$, $G(x, v - t^*)$ is increasing in its first argument and to the right it is decreasing. ■

Proof of Lemma 7. First, suppose that the population plays the equilibrium with no participation. Then the payoff to any level of violence is simply $-c(t)$ and the optimal choice is $t^* = 0$.

Now suppose that the population plays the cutoff equilibrium with cutoff $\hat{\theta}_H(v - t^*)$. Since the number of people who mobilize is simply the number of people whose type is greater than the cutoff rule, the probability of successful revolution is decreasing in $\hat{\theta}_H(v - t^*)$. Hence, it suffices to show that $\hat{\theta}_H(v - t^*)$ is increasing in t .

Implicitly differentiating Equation 3, we have that

$$\frac{\partial \hat{\theta}_H(v - t^*)}{\partial t} = - \frac{G_2(\hat{\theta}_H(v - t^*), v - t^*)}{G_1(\hat{\theta}_H(v - t^*), v - t^*)}.$$

The numerator is equal to $\frac{\partial G}{\partial v} \frac{\partial v}{\partial t}$, which is equal to $\phi(f(\hat{\theta})) \frac{\psi}{\sigma_2} \hat{\theta}_H(v - t^*)$, which is clearly positive. The denominator is negative by Lemma 6, which implies that $\frac{\partial \hat{\theta}_H(v - t^*)}{\partial t} > 0$. ■

Proof of Lemma 8.

Implicitly differentiating Equation 3, we have that

$$\frac{\partial \hat{\theta}_L(v - t^*)}{\partial \theta} = -\frac{G_2(\hat{\theta}_L(v - t^*), v - t^*)}{G_1(\hat{\theta}_L(v - t^*), v - t^*)} < 0.$$

The numerator is equal to $\frac{\partial G}{\partial v} \frac{\partial v}{\partial t}$, which is equal to $\phi(f(\hat{\theta})) \frac{\psi}{\sigma_2} \hat{\theta}_H(v - t^*)$, which is clearly positive. The denominator is positive by Lemma 6.

The proofs for $\hat{\theta}_L$ decreasing in the realization η and $t - t^*$ are identical, substituting $\frac{\partial v}{\partial \eta}$ or $\frac{\partial v}{\partial(t-t^*)}$ for $\frac{\partial v}{\partial \theta}$. ■

Proof of Lemma 9. From Proposition 1, an equilibrium with cutoff rule $\hat{\theta}_L$ exists at an information set if and only if a positive participation equilibrium exists if and only if

$$G(x^*(v - t^*), v - t^*) \geq k.$$

From Lemma 5, $G(x^*(v - t^*), v - t^*)$ is monotonically increasing in v .

From the definition of G it is immediate that $\lim_{v \rightarrow -\infty} G(x^*(v - t^*), v - t^*) = 0$. Since $v = \theta + \eta + t$, and θ and η have full support on the real line, v has full support on the real line. Thus, for sufficiently small realizations of $\eta + \theta$ there will not be a positive participation equilibrium. Moreover, since $G(x^*(v - t^*), v - t^*)$ is monotonically increasing and continuous in v , and since by Assumption 1 there exists some v such that $G(x^*(v - t^*), v - t^*) \geq k$, then there is some v where the inequality holds with equality and for any larger v it continues to hold strictly which establishes that a $\hat{\theta}(\eta + t - t^*)$ exists.

To see that $\hat{\theta}(\eta + t - t^*)$ is decreasing in η , notice that θ and η are substitutes in v and do not enter G anywhere else. ■

Proof of Lemma 10. The proof of the characterization of $\bar{\theta}(\eta + t - t^*)$ follows from the argument in the text. I now proceed to the two enumerated points.

1. $\bar{\theta}(\eta + t - t^*) = \hat{\theta}(\eta + t - t^*)$ if and only if

$$x^*(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*) - \Phi^{-1}(1 - T)\sigma_\epsilon \geq \hat{\theta}(\eta + t - t^*).$$

From Lemma 9, $\hat{\theta}(\eta + t - t^*)$ is decreasing in η . This implies that the right hand side of the above inequality is decreasing in η . Now consider the left-hand side.

$$\frac{\partial x^*(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*)}{\partial \eta} = \frac{\partial x^*(\hat{\theta}(\eta + t - t^*) + \eta + t - t^*)}{\partial v} \left(\frac{\partial \hat{\theta}}{\partial \eta} + 1 \right).$$

Now differentiating Equation 4, we have

$$\frac{\partial \hat{\theta}}{\partial \eta} = -\frac{G_1 \frac{\partial x^*}{\partial v} + G_2}{G_1 \frac{\partial x^*}{\partial v} + G_2} = -1.$$

Substituting back in yields $\frac{\partial x^*(\hat{\theta}(\eta+t-t^*)+\eta+t-t^*)}{\partial \eta} = 0$, so the left-hand side of the inequality is constant. Thus, the inequality holds for η sufficiently large. Label the minimal η as $\bar{\eta}(t-t^*)$, given by:

$$x^*(\hat{\theta}(\bar{\eta}+t-t^*)+\bar{\eta}+t-t^*) - \Phi^{-1}(1-T)\sigma_\epsilon = \hat{\theta}(\bar{\eta}+t-t^*).$$

2. From above, we have that if $\bar{\theta} = \hat{\theta}$ then it is decreasing ($\frac{\partial \hat{\theta}}{\partial \eta} = -1$.) Suppose instead that $\bar{\theta} = \theta^*$. Fix an η, t , and t^* and label $\eta+t-t^* = \kappa$. Now, if $\bar{\theta} = \theta^*$ we have

$$\hat{\theta}_L(\theta^*(\kappa) + \kappa) - \theta^*(\kappa) = \Phi^{-1}(1-T)\sigma_\epsilon.$$

Now increase η to $\eta' > \eta$ such that we have a new $\kappa' = \eta' + t - t^* > \kappa$. Label the distance between them $\Delta = \kappa' - \kappa$. Since, from Lemma 8, we have that $\hat{\theta}_L$ is decreasing in κ , we have that

$$\hat{\theta}_L(\theta^*(\kappa) + \kappa') - \theta^*(\kappa) < \Phi^{-1}(1-T)\sigma_\epsilon.$$

Further, we have that

$$\begin{aligned} \hat{\theta}_L((\theta^*(\kappa) - \Delta) + \kappa') - (\theta^*(\kappa) - \Delta) &= \hat{\theta}_L(\theta^*(\kappa) + \kappa) - \theta^*(\kappa) + \Delta \\ &> \Phi^{-1}(1-T)\sigma_\epsilon. \end{aligned}$$

Now, since $\hat{\theta}_L(v-t^*) - \theta$ is continuous in θ , the Intermediate Value Theorem implies that for some $\theta^*(\kappa') \in (\theta^*(\kappa) - \Delta, \theta^*(\kappa))$ we have

$$\hat{\theta}_L(\theta^*(\kappa') + \kappa') - \theta^*(\kappa') = \Phi^{-1}(1-T)\sigma_\epsilon.$$

Thus, $\theta^*(\eta+t-t^*)$ is decreasing in η .

■

Sketch of Proof of Lemma 11. Differentiating Equation 6, we have that an interior solution must satisfy the following first-order condition:

$$\begin{aligned}
& \frac{1}{\sigma_\theta \sigma_\eta} \left[- \int_{-\infty}^{\bar{\eta}(t^* - t^*)} \phi \left(\frac{\dot{\theta}(\tilde{\eta} + t^* - t^*)}{\sigma_\theta} \right) \frac{\partial \dot{\theta}(\tilde{\eta} + t^* - t^*)}{\partial t} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} \right. \\
& \quad + \left(\int_{\dot{\theta}(\bar{\eta}(t^* - t^*) + t^* - t^*)}^{\infty} \phi \left(\frac{\tilde{\theta}}{\sigma_\theta} \right) \phi \left(\frac{\bar{\eta}(t^* - t^*)}{\sigma_\eta} \right) d\tilde{\theta} \right) \frac{\partial \bar{\eta}(t^* - t^*)}{\partial t} \\
& \quad - \int_{\bar{\eta}(t^* - t^*)}^{\infty} \phi \left(\frac{\theta^*(\tilde{\eta} + t^* - t^*)}{\sigma_\theta} \right) \frac{\partial \theta^*(\tilde{\eta} + t^* - t^*)}{\partial t} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} \\
& \quad \left. - \left(\int_{\theta^*(\bar{\eta}(t^* - t^*) + t^* - t^*)}^{\infty} \phi \left(\frac{\tilde{\theta}}{\sigma_\theta} \right) \phi \left(\frac{\bar{\eta}(t^* - t^*)}{\sigma_\eta} \right) d\tilde{\theta} \right) \frac{\partial \bar{\eta}(t^* - t^*)}{\partial t} \right] - c'(t^*) = 0
\end{aligned}$$

I use the following facts:

1. From the proof of Lemma 10, $\frac{\partial \dot{\theta}(x)}{\partial x} = -1$ for all x .
2. $\phi \left(\frac{\bar{\eta}(t^* - t^*)}{\sigma_\eta} \right)$ does not depend on the $\tilde{\theta}$ over which we are integrating in the first and fourth terms of the first-order condition
3. In equilibrium, beliefs about t are correct.

Given these fact, we can rewrite the first-order condition as follows:

$$\begin{aligned}
& \frac{1}{\sigma_\theta \sigma_\eta} \left[\int_{-\infty}^{\bar{\eta}(0)} \phi \left(\frac{\dot{\theta}(\tilde{\eta})}{\sigma_\theta} \right) \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} - \int_{\bar{\eta}(0)}^{\infty} \phi \left(\frac{\theta^*(\tilde{\eta})}{\sigma_\theta} \right) \frac{\partial \theta^*(\tilde{\eta})}{\partial t} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} \right. \\
& \quad \left. + \phi \left(\frac{\bar{\eta}(0)}{\sigma_\eta} \right) \frac{\partial \bar{\eta}(0)}{\partial t} \left(\Phi \left(\frac{\theta^*(\bar{\eta}(0))}{\sigma_\theta} \right) - \Phi \left(\frac{\dot{\theta}(\bar{\eta}(0))}{\sigma_\theta} \right) \right) \right] = c'(t^*).
\end{aligned}$$

By the definition of $\bar{\eta}$, $\theta^*(\bar{\eta}(0)) = \dot{\theta}(\bar{\eta}(0))$, so the third term on the left-hand side is equal to 0. Thus, the first-order condition reduces to

$$\frac{1}{\sigma_\theta \sigma_\eta} \left[\int_{-\infty}^{\bar{\eta}(0)} \phi \left(\frac{\dot{\theta}(\tilde{\eta})}{\sigma_\theta} \right) \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} - \int_{\bar{\eta}(0)}^{\infty} \phi \left(\frac{\theta^*(\tilde{\eta})}{\sigma_\theta} \right) \frac{\partial \theta^*(\tilde{\eta})}{\partial t} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} \right] = c'(t^*).$$

Now, since c is strictly convex, this equation has at most one solution, and it has at least one since $c'(\cdot)$ satisfies the Inada conditions.

Note, it remains to be shown that t^* is in fact a best response by the vanguard. To do so, it suffices to establish the concavity of the objective at the critical value. Taking the second derivative shows that the objective is concave at the solution as long as a finite constant is less than $c''(t)$. Since $c''' \geq 0$, there exists an \bar{c} such that, if $c''(0) \geq \bar{c}$, the objective is concave. It remains to characterize \bar{c} in terms of primitives. ■

Proof of Proposition 2. Follows from Proposition 1, Lemma 7, and Lemma 11. ■

Proof of Proposition 3. I make use of the following lemmata.

Lemma 12 $\hat{\theta}_L(\theta + \eta)$ is

1. *Increasing in T*
2. *Increasing in k*
3. *Decreasing in γ*

Proof of Lemma 12.

$\hat{\theta}_L(\theta + \eta)$ is implicitly defined as the lowest $\hat{\theta}$ that satisfies Equation 3. Implicitly differentiating (and dropping the functional dependence of $\hat{\theta}_L$ to avoid clutter) we have:

1.
$$\frac{\partial \hat{\theta}_L}{\partial T} = \frac{\phi(\alpha \hat{\theta}_L - \beta) \gamma \hat{\theta}_L \frac{\sigma_z}{\sigma_2} (\Phi^{-1})'(1 - T)}{G_1(\hat{\theta}_L, v - t^*)} > 0,$$

where the inequality follows from the facts that the numerator is clearly positive and the denominator is positive by Lemma 6.

2.
$$\frac{\partial \hat{\theta}_L}{\partial k} = \frac{1}{G_1(\hat{\theta}_L, v - t^*)} > 0,$$

where the inequality again follows from the fact that the denominator is positive by Lemma 6.

3.
$$\frac{\partial \hat{\theta}_L}{\partial \gamma} = \frac{-(1 - \Phi(\alpha \hat{\theta}_L - \beta)) \hat{\theta}_L}{G_1(\hat{\theta}_L, v - t^*)} < 0,$$

where the inequality follows from the fact that the numerator is clearly negative and the denominator is positive by Lemma 6.

■

Lemma 13 $\hat{\theta}(\eta)$ is

1. *Increasing in T*
2. *Increasing in k*
3. *Decreasing in γ*

Proof of Lemma 13.

First note that $\hat{\theta}(\eta)$ is implicitly defined by Equation 4. Thus we implicitly differentiate this equation to derive the results.

1.

$$\frac{\partial \hat{\theta}}{\partial T} = - \frac{G_1(x^*(\hat{\theta} + \eta), v - t^*) \frac{\partial x^*(\hat{\theta} + \eta)}{\partial T} - \phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\sigma_\epsilon}{\sigma_2} (\phi^{-1})'(1 - T) \gamma x^*(\hat{\theta} + \eta)}{G_1(x^*(\hat{\theta} + \eta), v - t^*) \frac{\partial x^*(\hat{\theta} + \eta)}{\partial \theta} + \phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)}$$

Notice that since $x^*(\hat{\theta} + \eta)$ is a maximizer of G with respect to x , we have that $G_1(x^*(\hat{\theta} + \eta), v - t^*) = 0$. Thus, we can rewrite the derivative above as

$$\frac{\partial \hat{\theta}}{\partial T} = \frac{\phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\sigma_\epsilon}{\sigma_2} (\phi^{-1})'(1 - T) \gamma x^*(\hat{\theta} + \eta)}{\phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)} > 0,$$

where the inequality follows from the fact that x^* has to be positive and $(\Phi^{-1})'$ is clearly positive since Φ is strictly increasing.

2.

$$\frac{\partial \hat{\theta}}{\partial k} = - \frac{-1}{G_1(x^*(\hat{\theta} + \eta), v - t^*) \frac{\partial x^*(\hat{\theta} + \eta)}{\partial \theta} + \phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)}$$

Again using the fact that $G_1(x^*(\hat{\theta} + \eta), v - t^*) = 0$, we can rewrite this as

$$\frac{\partial \hat{\theta}}{\partial k} = \frac{1}{\phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)} > 0.$$

3.

$$\frac{\partial \hat{\theta}}{\partial \gamma} = - \frac{G_1(x^*(\hat{\theta} + \eta), v - t^*) \frac{\partial x^*(\hat{\theta} + \eta)}{\partial \gamma} + (1 - \Phi(\alpha x^*(\hat{\theta} + \eta) - \beta)) x^*(\hat{\theta} + \eta)}{G_1(x^*(\hat{\theta} + \eta), v - t^*) \frac{\partial x^*(\hat{\theta} + \eta)}{\partial \theta} + \phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)}.$$

Again using the fact that $G_1(x^*(\hat{\theta} + \eta), v - t^*) = 0$, we can rewrite this as

$$\frac{\partial \hat{\theta}}{\partial \gamma} = - \frac{(1 - \Phi(\alpha x^*(\hat{\theta} + \eta) - \beta)) x^*(\hat{\theta} + \eta)}{\phi(\alpha x^*(\hat{\theta} + \eta) - \beta) \frac{\psi}{\sigma_2} \gamma x^*(\hat{\theta} + \eta)} < 0.$$

■

The number (measure) of people who mobilize is given by

$$N = \int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_L(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta}.$$

Differentiating we have

1.

$$\begin{aligned} \frac{\partial N}{\partial T} &= \int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} -\phi \left(\frac{\hat{\theta}_L(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial T} \frac{1}{\sigma_\epsilon} \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta} \\ &\quad - \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_L(\hat{\theta}(\tilde{\eta}) + \tilde{\eta}) - \hat{\theta}(\tilde{\eta})}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\hat{\theta}(\tilde{\eta}) - m}{\sigma_\theta} \right) \frac{\partial \hat{\theta}(\tilde{\eta})}{\partial T} \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} < 0 \end{aligned}$$

where the inequality follows from the fact that $\frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial T} > 0$ and $\frac{\partial \hat{\theta}(\tilde{\eta})}{\partial T} > 0$ from Lemmata 12 and 13, respectively. Thus, as T increases, mobilization decreases and the number of people needed for victory increases, so the probability of victory also decreases.

2.

$$\begin{aligned} \frac{\partial N}{\partial k} &= \int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} -\phi \left(\frac{\hat{\theta}_L(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial k} \frac{1}{\sigma_\epsilon} \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta} \\ &\quad - \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_L(\hat{\theta}(\tilde{\eta}) + \tilde{\eta}) - \hat{\theta}(\tilde{\eta})}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\hat{\theta}(\tilde{\eta}) - m}{\sigma_\theta} \right) \frac{\partial \hat{\theta}(\tilde{\eta})}{\partial k} \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} < 0 \end{aligned}$$

where the inequality follows from the fact that $\frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial k} > 0$ and $\frac{\partial \hat{\theta}(\tilde{\eta})}{\partial k} > 0$ from Lemmata 12 and 13, respectively. Thus, as k increases, mobilization decreases and T stays constant, so the probability of victory also decreases.

3.

$$\begin{aligned} \frac{\partial N}{\partial \gamma} &= \int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} -\phi \left(\frac{\hat{\theta}_L(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial \gamma} \frac{1}{\sigma_\epsilon} \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta} \\ &\quad - \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_L(\hat{\theta}(\tilde{\eta}) + \tilde{\eta}) - \hat{\theta}(\tilde{\eta})}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\hat{\theta}(\tilde{\eta}) - m}{\sigma_\theta} \right) \frac{\partial \hat{\theta}(\tilde{\eta})}{\partial \gamma} \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\eta} > 0 \end{aligned}$$

where the inequality follows from the fact that $\frac{\partial \hat{\theta}_L(\tilde{\theta} + \tilde{\eta})}{\partial \gamma} < 0$ and $\frac{\partial \hat{\theta}(\tilde{\eta})}{\partial \gamma} < 0$ from Lemmata 12 and 13, respectively. Thus, as γ increases, mobilization increases and T stays constant, so the probability of victory also increases.

■

Proof of Proposition 4. Follows immediately from the second enumerated point of Lemma 10.

■

Proof of Proposition 5.

As established in Proposition 2, the vanguard engages in non-minimal effort if the population plays the cutoff rule $\hat{\theta}_L$ and does not engage in effort if the population plays the cutoff rule $\hat{\theta}_H$.

1. The expected level of mobilization on the two paths are

$$\int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_L(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta}$$

and

$$\int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \left(1 - \Phi \left(\frac{\hat{\theta}_H(\tilde{\theta} + \tilde{\eta}) - \tilde{\theta}}{\sigma_\epsilon} \right) \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta} - m}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\theta} d\tilde{\eta},$$

respectively. It is obvious from the fact that $\hat{\theta}_H > \hat{\theta}_L$, that the latter is smaller than the former.

2. Probability of success is monotonic in expected mobilization, so this point follows directly from the previous point.
3. Assuming a positive participation equilibrium exists, a member of the population mobilizes if $\epsilon_i \geq \hat{\theta} - \theta$. Thus, for a fixed θ and η where an equilibrium $\hat{\theta}$ exists, the average level of extremism among participants is:

$$\int_{\hat{\theta}(\theta+\eta)-\theta}^{\infty} (\theta + \tilde{\epsilon}) \frac{1}{\sigma_\epsilon} \phi \left(\frac{\tilde{\epsilon}}{\sigma_\epsilon} \right) d\epsilon.$$

Of course, a positive participation equilibrium only exists if $\theta \geq \hat{\theta}(\eta)$. So integrating this expected extremism over all (θ, η) pairs where a positive participation equilibrium exists, we get that the expected extremism among participants on the path where a vanguard engages in non-minimal effort is given by:

$$\int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \int_{\hat{\theta}_L(\tilde{\theta}+\tilde{\eta})-\tilde{\theta}}^{\infty} (\tilde{\theta} + \tilde{\epsilon}) \frac{1}{\sigma_\epsilon} \phi \left(\frac{\tilde{\epsilon}}{\sigma_\epsilon} \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta}}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\epsilon} d\tilde{\theta} d\tilde{\eta},$$

and on the path where a vanguard engages in minimal effort is given by:

$$\int_{-\infty}^{\infty} \int_{\hat{\theta}(\tilde{\eta})}^{\infty} \int_{\hat{\theta}_H(\tilde{\theta}+\tilde{\eta})-\tilde{\theta}}^{\infty} (\tilde{\theta} + \tilde{\epsilon}) \frac{1}{\sigma_\epsilon} \phi \left(\frac{\tilde{\epsilon}}{\sigma_\epsilon} \right) \frac{1}{\sigma_\theta} \phi \left(\frac{\tilde{\theta}}{\sigma_\theta} \right) \frac{1}{\sigma_\eta} \phi \left(\frac{\tilde{\eta}}{\sigma_\eta} \right) d\tilde{\epsilon} d\tilde{\theta} d\tilde{\eta}.$$

The fact that the former is smaller than the latter follows directly from the fact that $\hat{\theta}_H > \hat{\theta}_L$, so the expectation of ϵ on $\hat{\theta}_H$ to infinity is larger than on $\hat{\theta}_L$ to infinity.

■

Proof of Proposition 6.

An analysis identical to the one in Lemma 1 shows that in this game there is always an equilibrium without participation.

Following a signal s_i a player has beliefs about θ that are normally distributed with mean $\bar{m}_i = \lambda s_i + (1 - \lambda)m$ and variance $\lambda\sigma_\epsilon^2$. If all players play a cutoff rule that involves joining the revolution if and only if $s_i \geq \hat{s}$, then, just as before, the revolution will be successful as long as $\theta \geq \theta^*$ with $\theta^* = \hat{s} - \Phi^{-1}(1 - T)\sigma_\epsilon$. From the perspective of a person who receives the signal s_i , the probability of victory is $1 - \Phi\left(\frac{\theta^* - \bar{m}_i}{\sigma_\epsilon\sqrt{\lambda}}\right)$. Such a person will participate if

$$\left(1 - \Phi\left(\frac{\theta^* - \bar{m}_i}{\sigma_\epsilon\sqrt{\lambda}}\right)\right)\gamma\bar{m} \geq k,$$

which can be rewritten:

$$\left(1 - \Phi\left(\frac{\hat{s} - \Phi^{-1}(1 - T)\sigma_\epsilon - \lambda s_i - (1 - \lambda)m}{\sigma_\epsilon\sqrt{\lambda}}\right)\right)\gamma(\lambda s_i + (1 - \lambda)m) \geq k.$$

The left-hand side is clearly increasing in s_i , so if there is a cutoff rule, it must satisfy:

$$\left(1 - \Phi\left(\frac{(1 - \lambda)\hat{s} - \Phi^{-1}(1 - T)\sigma_\epsilon - (1 - \lambda)m}{\sigma_\epsilon\sqrt{\lambda}}\right)\right)\gamma(\lambda\hat{s} + (1 - \lambda)m) = k.$$

Now, it is straightforward to verify that Lemmata 2–4 hold for the left-hand side of this condition, with essentially identical proofs. This implies that the qualitative results of Proposition 1 and ?? continue to hold. ■

Proof of Proposition 7.

The proof follows exactly the proof in Angeletos, Hellwig and Pavan (2007) and is included only for completeness.

For a given cutoff rule \hat{t} an individual will participate if $\xi_i \leq \hat{t} - T$. The number of participants is $\Phi\left(\frac{\hat{t} - T}{\sigma_\xi}\right)$ and there is victory if T is less than $T^*(\hat{t})$ given by:

$$\Phi\left(\frac{\hat{t} - T^*}{\sigma_\xi}\right) = T^*$$

which can be rewritten

$$\hat{t} = \Phi^{-1}(T^*)\sigma_\xi + T^*. \tag{12}$$

A person who observes signal t_i has posteriors that are normally distributed with mean $m_i = \beta t_i + (1 - \beta)m$ and variance $\sigma^2 = \beta\sigma_\xi^2$, with $\beta = \frac{\sigma_t^2}{\sigma_t^2 + \sigma_\xi^2}$. Thus, a person with signal t_i believes that the probability of victory, given cutoff rule \hat{t} is

$$\Phi\left(\frac{T^*(\hat{t}) - \bar{m}_i}{\sigma}\right).$$

She will participate if

$$\Phi\left(\frac{T^*(\hat{t}) - \beta t_i - (1 - \beta)m}{\sigma}\right) \gamma \theta \geq k.$$

The left-hand side is decreasing in t_i so a cutoff rule must satisfy

$$\Phi\left(\frac{T^*(\hat{t}) - \beta \hat{t} - (1 - \beta)m}{\sigma}\right) \gamma \theta = k,$$

which, substituting from Equation 12 can be rewritten

$$\Phi\left(\frac{(1 - \beta)T^*(\hat{t}) - \beta\Phi^{-1}(T^*(\hat{t}))\sigma_\xi - (1 - \beta)m}{\sigma}\right) \gamma \theta = k. \quad (13)$$

Differentiating the right-hand side of Equation 13 with respect to \hat{t} yields

$$\frac{\partial RHS}{\partial \hat{t}} = \phi\left(\frac{(1 - \beta)T^*(\hat{t}) - \beta\Phi^{-1}(T^*(\hat{t}))\sigma_\xi - (1 - \beta)m}{\sigma}\right) \frac{\gamma \theta}{\sigma} \frac{\partial T^*}{\partial \hat{t}} \left[(1 - \beta) - \frac{\beta \sigma_\xi}{\phi(\Phi^{-1}(T^*(\hat{t})))} \right].$$

Implicitly differentiating Equation 12 shows that $\frac{\partial T^*}{\partial \hat{t}} = \frac{\phi\left(\frac{T^*(\hat{t}) - \bar{m}_i}{\sigma}\right)}{\phi\left(\frac{T^*(\hat{t}) - \bar{m}_i}{\sigma}\right) + 1} > 0$, so the derivative of the right-hand side of Equation 13 is negative if and only if $(1 - \beta) - \frac{\beta \sigma_\xi}{\phi(\Phi^{-1}(T^*(\hat{t})))} < 0$. Substituting for β and using the fact that $\min_x \frac{1}{\phi(\Phi^{-1}(x))} = \sqrt{2\pi}$, the right-hand side of Equation 13 is monotonically decreasing if $\sigma_\xi < \sigma_t^2 \sqrt{2\pi}$. When this condition holds the game clearly has a unique cutoff rule consistent with equilibrium. ■

Appendix B: Stability of Equilibria

Section 4 shows that a static, exogenous information version of my model generates three equilibria in cutoff strategies—one with no participation and two with positive participation. Since the global games information structure was designed as a type of perturbation to create equilibrium selection, it is important to confirm that these multiple equilibria are robust to other standard types of perturbations or stability requirements.

This issue is particularly salient because the positive participation equilibrium with the higher threshold may seem intuitively unstable. This intuition comes from thinking about a tatonnement adjustment process. The idea is that if, for some reason, players are knocked out of the $\hat{\theta}_H$ equilibrium, then best responses do not call for players to return to that equilibrium. In particular, fix a θ' between $\hat{\theta}_L$ and $\hat{\theta}_H$. Now, suppose that players were supposed to play the higher cutoff, but a set of players with types lying in $[\theta', \hat{\theta})$ accidentally participate. Then players with types just lower than θ' can make positive profits by participating (this is clear from Figure 1), so one would expect them to participate. This will continue to be true until everyone down to $\hat{\theta}_L$ participates. Similarly, fix a finite θ' greater than $\hat{\theta}_H$. If a set of players with types lying in $[\hat{\theta}, \theta']$ do not participate

by accident, then players with types just above θ' make negative profits and so would rather not participate, which will then continue to be true for ever higher types, leading to no participation. On this sort of evolutionary instability argument, one might be tempted to rule out the higher positive participation equilibrium. If one does, the game still has multiple equilibria, but only one with positive participation.

More importantly, however, standard static notions of stability do not rule out either of the positive participation equilibria. In what follows, I show that both positive participation equilibria are stable under perturbations of the threshold T and perturbations to the behavior of sets of players (see Myerson (1991, Chapter 5) for a discussion of these types of stability criteria).

Perturbations to the Threshold

For this analysis I consider equilibria of the game characterized by the revolution stage ignoring the vanguard stage (i.e., the revolution game without endogenous information). I refer to this game as the “unperturbed game” and continue to denote it Γ . (The same arguments would extend easily to the full game.)

I consider an equilibrium of the unperturbed game stable if it is the limit of a sequence of equilibria to a sequence of perturbed games as the perturbation goes to zero. Consider the following game with *threshold perturbations*. In this variant of the game, the threshold is a random variable X_T distributed according to the uniform distribution on $[T - \iota, T + \iota]$. I notate the perturbed games Γ^ι . Refer to a particular realization of X_T as t . The notion of stability conforms to the following notion of convergence.

Definition 2 *A cutoff equilibrium in the unperturbed game with cutoff rule $\hat{\theta}$ is threshold convergent if there is a sequence of games with threshold perturbations and cutoff equilibria in those perturbed games, $\{\Gamma^\iota, \hat{\theta}^\iota\}$, such that for every $\delta > 0$ there exists an $\iota(\delta)$ satisfying $|\hat{\theta}^\iota - \hat{\theta}| < \delta$ whenever $\iota < \iota(\delta)$.*

This definition allows me to establish the following result.

Proposition 8 *Any cutoff equilibrium of the unperturbed game with positive participation and cutoff rule $\hat{\theta}$ is threshold convergent.*

Proof of Proposition 8. Just as in the unperturbed game, in the perturbed game, for a given cutoff rule $\hat{\theta}^\iota$ and a realization of X_T , t , we can define the minimal level of support needed for victory as:

$$\theta^*(t) = \hat{\theta}^\iota - \Phi^{-1}(1 - t)\sigma_\epsilon.$$

From the perspective of person i , the probability of victory, conditional on θ_i is:

$$\int_{T-\iota}^{T+\iota} \left(1 - \Phi \left(\frac{\theta^*(\tilde{t}) - \bar{m}}{\bar{\sigma}_\theta} \right) \right) \frac{1}{2\iota} d\tilde{t}.$$

Substituting and rearranging, such a person will participate if:

$$\theta_i \int_{T-\iota}^{T+\iota} \left(1 - \Phi \left(\frac{(1-\lambda)\theta_i - \frac{(1-\lambda)m + \sigma_\epsilon \Phi^{-1}(1-\tilde{t})}{\sigma_\epsilon \sqrt{\lambda}}}{\sigma_\epsilon \sqrt{\lambda}} \right) \right) \frac{1}{2\iota} d\tilde{t} \geq k.$$

The left-hand side of this condition is clearly increasing in θ_i and so we have a cutoff equilibrium for any $\hat{\theta}^\iota$ satisfying:

$$\hat{\theta}^\iota \int_{T-\iota}^{T+\iota} \left(1 - \Phi \left(\frac{(1-\lambda)\hat{\theta}^\iota - \frac{(1-\lambda)m + \sigma_\epsilon \Phi^{-1}(1-\tilde{t})}{\sigma_\epsilon \sqrt{\lambda}}}{\sigma_\epsilon \sqrt{\lambda}} \right) \right) \frac{1}{2\iota} d\tilde{t} = k.$$

Label the LHS of this condition $\hat{G}(x, \iota)$, where the definition of \hat{G} is analogous to G in the original model. Notice, first that $\hat{G}(x, \iota)$ has all the same properties that Lemma 4 establishes for G and that $\lim_{\iota \rightarrow 0} \hat{G}(x, \iota) = G(x)$ for all x . Further $\hat{G}(x, \iota)$ is continuous in ι on the positive reals.

Now, consider a particular cutoff equilibrium of the unperturbed game $\hat{\theta}'$. Since, \hat{G} is continuous in ι and $\hat{G}(x, 0) = G(x)$, we have that $\hat{\theta}' \in \hat{\theta}^0$ and the Implicit Function Theorem implies that $\hat{\theta}^\iota$ is continuous in ι in a neighborhood around $\iota = 0$. Now, by the definition of continuity, for any $\delta > 0$, we can find an ι sufficiently close to 0 that there is a $\hat{\theta}^\iota$ satisfying $|\hat{\theta}^\iota - \hat{\theta}'| < \delta$, establishing threshold convergence. ■

Perturbations to Behavior

Here I consider a game with *behavior perturbations* in which each player has a small probability $\nu < \min\{\frac{1}{2}, 1-T\}$ of making a mistake (i.e., of choosing to participate when she intended not to participate or vice-versa). I notate these perturbed games by Γ^ν . The notion of stability will conform to the following notion of convergence.

Definition 3 *A cutoff equilibrium in the unperturbed game with cutoff rule $\hat{\theta}$ is behavior convergent if there is a sequence of games with behavior perturbations and cutoff equilibria in those perturbed games, $\{\Gamma^\nu, \hat{\theta}^\nu\}$, such that for every $\delta > 0$ there exists an $\nu(\delta)$ satisfying $|\hat{\theta}^\nu - \hat{\theta}| < \delta$ whenever $\nu < \nu(\delta)$.*

Now I have the following result.

Proposition 9 *Any cutoff equilibrium of the unperturbed game with positive participation and cutoff rule $\hat{\theta}$ is behavior convergent.*

Proof. Just as in the unperturbed game, in the perturbed game, for a given cutoff rule $\hat{\theta}^\nu$, we can calculate the level of participation for any given θ as:

$$\left(1 - \Phi \left(\frac{\hat{\theta}^\nu - \theta}{\sigma_\epsilon} \right) \right) (1 - \nu) + \Phi \left(\frac{\hat{\theta}^\nu - \theta}{\sigma_\epsilon} \right) (1 - \nu).$$

Then, the level of support needed for victory is

$$\theta^*(\nu) = \hat{\theta}^\nu - \Phi^{-1} \left(\frac{1 - T - \nu}{1 - 2\nu} \right) \sigma_\epsilon.$$

Thus, from the perspective of person i , the probability of victory, conditional on θ_i is

$$1 - \Phi \left(\frac{\theta^*(\nu) - \bar{m}}{\bar{\sigma}_\theta} \right),$$

just as in the unperturbed game (except the definition of θ^* is different). Such a person will choose to participate if

$$\left[1 - \Phi \left(\frac{\theta^*(\nu) - \bar{m}}{\bar{\sigma}_\theta} \right) \right] \theta_i \geq k.$$

Since \bar{m} is clearly increasing in θ_i , the LHS of this inequality is increasing in θ_i . Thus, we can define a threshold $\hat{\theta}^\nu$ such that a person participates if $\theta_i \geq \hat{\theta}^\nu$. Substituting for \bar{m} , this threshold is given by:

$$\left[1 - \Phi \left(\frac{\theta^*(\nu) - \lambda \hat{\theta}^\nu - (1 - \lambda)m}{\bar{\sigma}_\theta} \right) \right] \hat{\theta}^\nu = k.$$

Substituting for $\bar{\sigma}_\theta$ and $\theta^*(\nu)$, we can express an equilibrium in the perturbed game in one condition:

$$\left[1 - \Phi \left(\frac{(1 - \lambda) \hat{\theta}^\nu - \frac{(1 - \lambda)m + \sigma_\epsilon \Phi^{-1} \left(\frac{1 - T - \nu}{1 - 2\nu} \right)}{\sigma_\epsilon \sqrt{\lambda}}}{\sigma_\epsilon \sqrt{\lambda}} \right) \right] \hat{\theta}^\nu = k. \quad (14)$$

Label the LHS of this condition $\tilde{G}(x, \nu)$, where the definition of \tilde{G} is analogous to G in the original model. Notice, first that $\tilde{G}(x, \nu)$ has all the same properties that Lemma 4 establishes for G and that $\lim_{\nu \rightarrow 0} \tilde{G}(x, \nu) = G(x)$ for all x . Further $\tilde{G}(x, \nu)$ is continuous in ν on the positive reals.

Now, consider a particular cutoff equilibrium of the unperturbed game $\hat{\theta}'$. Since, \tilde{G} is continuous in ν and $\tilde{G}(x, 0) = G(x)$, we have that $\hat{\theta}' \in \hat{\theta}^0$ and the Implicit Function Theorem implies that $\hat{\theta}^\nu$ is continuous in ν in a neighborhood around $\nu = 0$. Now, by the definition of continuity, for any $\delta > 0$, we can find an ν sufficiently close to 0 that there is a $\hat{\theta}^\nu$ satisfying $|\hat{\theta}^\nu - \hat{\theta}'| < \delta$, establishing behavior convergence. ■

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