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Interval Properties of Ideal Point Estimators

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by

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Abstract

This paper is a preliminary exploration of the problems posed by low levels of voting error in the recovery of interval level parameter estimates in spatial (geometric) models of parliamentary voting. Our results, though limited, show strong support for the Quinn Conjecture, namely, if the voting space is one dimensional and the noise process is symmetric, then as the number of roll calls goes to infinity the true rank ordering of the legislators will be recovered.

We also discuss the “sag” problem – errorless voting by a legislator at the end of the dimension – and how it relates to estimating ideal point configurations for the United States Supreme Court.

1. Introduction

In the past 20 years spatial (geometric) models of choice have become an important area of theoretical and empirical research in Political Science. The literature has grown exponentially and the spatial model has proven to be remarkably successful in explaining and *predicting* the choice behavior of political actors.[1]

We focus on the standard spatial model of parliamentary roll call voting in one dimension. Our focus is the interaction of the level of error and the interval properties of the parameters of the model. In one dimension, Poole (2005, ch. 2) has proven that if voting is "perfect" (no voting errors) then the legislator configuration is identified only up to a weak monotone transformation of the true rank ordering. If there are roll call calls that divide every adjacent pair of legislators then the true rank ordering is recovered.[2] This raises a fundamental issue: how high does the error level have to be for an *interval level* legislator configuration to be *reliably* recovered from the roll calls. This paper is a very preliminary investigation of this problem.

In the next section we briefly outline the standard spatial model of parliamentary voting in one dimension. We then discuss the "sag" problem – legislators at the ends of the dimension who are perfect voters – and how it is related to the general problem of interval level ideal point estimates. We use the United States Supreme Court as an example of this problem.

2. The One Dimensional Spatial Model of Parliamentary Roll Call Voting

Suppose there are p legislators and q roll calls. We assume that each legislator has an ideal point on the dimension represented by the point X_i with a symmetric singlepeaked utility function centered at the ideal point. With no error and sincere voting, the legislator votes for the closest alternative in the policy space on every roll call because her utility function is *symmetric*. Let the two policy outcomes corresponding to Yea and Nay on the jth roll call be represented by O_{jy} and O_{jn} respectively. In most cases it is convenient to work with the midpoint of the two outcomes:

$$Z_j = \frac{O_{jy} + O_{jn}}{2} \qquad (1)$$

In one dimension Z_j is known as a *cutting point* that divides the Yeas from the Nays. With perfect spatial voting all the legislators to the left of Z_j vote for one outcome and all the legislators to the right of Z_j vote for the opposite outcome.

The major probabilistic models of parliamentary voting are based on the *random utility model* developed by McFadden (1976). In the random utility model, a legislator's overall utility for voting Yea is the sum of a deterministic utility and a random error. The same is true for the utility of voting Nay. Legislator i's utility for the Yea outcome on roll call j is:

$$U_{ijy} = u_{ijy} + \varepsilon_{ijy} \qquad (2)$$

where u_{ijy} is the *deterministic* portion of the utility function and ε_{ijy} is the *stochastic* or *random* portion of the utility function. The two most widely used deterministic utility functions are the normal;

$$u_{ijy} = \beta e^{\left(-\frac{1}{2}d_{ijy}^2\right)} \qquad (3)$$

and the quadratic;

$$u_{ijy} = -d_{ijy}^2 \qquad (4)$$

where d^2_{ijy} is the squared distance of the i^{th} legislator to the Yea outcome;

$$d^2_{ijy} = (X_i - O_{jy})^2$$

and β is an overall signal–to-noise parameter (Poole, 2005; Poole and Rosenthal, 1997, 2006).

If there is no error, then legislator i votes Yea if $U_{ijy} > U_{ijn}$. Equivalently, if the difference, $U_{ijy} - U_{ijn}$, is positive, then legislator i votes Yea. With random error the utility difference is:

$$U_{ijy} - U_{ijn} = u_{ijy} - u_{ijn} + \varepsilon_{ijy} - \varepsilon_{ijn}$$

So that the legislator votes Yea if:

$$u_{ijy} - u_{ijn} > \varepsilon_{ijn} - \varepsilon_{ijy}$$

That is, the legislator votes Yea if the difference in the deterministic utilities is greater than the difference between the two random errors. Because the errors are unobserved, we must make an assumption about the error distribution from which they are drawn. Armed with that assumption, we can calculate the *probability* that the legislator will vote Yea.

We assume that ε_{ijn} and ε_{ijy} are drawn (a random sample of size two) from a normal distribution with mean zero and variance one-half. The difference between the two errors has a standard normal distribution; that is

$$\varepsilon_{ijn} - \varepsilon_{ijy} \sim N(0, 1)$$

and the distribution of the difference between the overall utilities is

$$U_{ijy} - U_{ijn} \sim N(u_{ijy} - u_{ijn}, 1)$$

Hence the probability that legislator i votes Yea on the j^{th} roll call can be rewritten as:

$$P_{ijy} = P(U_{ijy} > U_{ijn}) = P(\varepsilon_{ijn} - \varepsilon_{ijy} < u_{ijy} - u_{ijn}) = \Phi[u_{ijy} - u_{ijn}] \qquad (5)$$

For the normal deterministic utility function the probability of voting Yea becomes:

$$P_{ijy} = \Phi\left[\beta\left\{e^{\left(-\frac{1}{2}d_{ijy}^2\right)} - e^{\left(-\frac{1}{2}d_{ijkn}^2\right)}\right\}\right] \tag{6}$$

Hereafter we refer to this as the *Normal-Normal* (NN) model.

For the quadratic deterministic utility function the probability of voting Yea becomes:

$$P_{ijy} = \Phi\left[-d_{ijy}^2 + d_{ijkn}^2\right] \tag{7}$$

Hereafter we refer to this as the *Quadratic-Normal* (QN) model.

Equation (7) can be easily simplified (Martin and Quinn, 2002; Clinton, Jackman, and Rivers, 2004; Poole, 2005). In terms of our notation:

$$\begin{array}{l} \text{If } O_{jn} > O_{jy} \text{ then } P_{ijy} = 2(O_{jn} - O_{jy})(\mathbf{Z_j} - X_i) \\ \text{If } O_{jy} > O_{jn} \text{ then } P_{ijy} = 2(O_{jy} - O_{jn})(\mathbf{Z_j} - X_i) \end{array} \tag{8}$$

3. The Sag Problem

If voting is "perfect" (no voting errors) in one dimension and there are cutting points -- $\mathbf{Z_j}$ -- between every adjacent pair of legislators then the true rank ordering of the legislators is recovered. Unfortunately, there is (as yet) no solution for the perfect voting problem in more than one dimension (Poole, 2005). The perfect voting problem goes beyond the obvious point that with no error probabilistic voting models are inappropriate (if there is no error there is no probability!). To illustrate, consider the standard ordinary least squares regression model. Given interval level variables ordinary least squares regression analysis will yield interval level coefficients regardless the level of underlying error because the coefficients are mathematical functions of the principle of least squares. There is no counterpart to the principle of least squares for discrete choice models. Put simply: if there is no error, both probit and logit "blow up" (that is, the maximum likelihood estimator of the parameters does not exist [Silvapulle, 1981]). This problem is known as *complete separation* (Silvapulle, 1981; Albert and Anderson, 1984).

In practice, perfect data is usually not observed (see our discussion below). However, it is not uncommon to find legislators at the ends of the dimension who are *perfect liberals* or *perfect conservatives*; that is, either they vote for the conservative outcome on nearly every roll call or they vote for the liberal outcome on nearly every roll call. Consequently, a "sag" in the ideal point configuration can occur, in that a legislator appears at the edge of the space with the other legislators recovered to the interior.

The sag problem can be especially serious if probabilistic methods are used on small voting bodies, such as the U. S. Supreme Court. For example, Table 1 [see Poole (2005, p. 157)] shows the recovered one-dimensional ideal points of the recent Rehnquist court if we use W-NOMINATE with no sag constraint, Quadratic-Normal Scaling (Poole, 2001) with no sag constraints, optimal classification (OC) one-dimensional rank ordering (Poole, 2000), and OC two-dimensional coordinates.

Table 1: The Supreme Court and the Sag Problem

	W-NOMINATE	Quad-Norm Scaling	OC	OC(1)	OC(2)
STEVENS	-1.000	-1.000	1.000	-0.794	-0.491
BREYER	-0.454	-0.644	2.000	-0.576	0.906
GINSBURG	-0.302	-0.498	3.000	-0.357	-0.357
SOUTER	-0.175	-0.237	4.000	-0.270	-0.078
KENNEDY	0.273	0.210	5.000	0.160	-0.287
OCONNOR	0.319	0.381	6.000	0.194	0.790
REHNQUIST	0.558	0.675	7.000	0.416	0.022
SCALIA	0.782	0.895	8.000	0.554	-0.309
THOMAS	1.000	1.000	9.000	0.593	-0.211

All three methods produce the same rank ordering of the 9 Justices. Both W-NOMINATE and Quadratic-Normal Scaling, however, show a considerable gap between Stevens and Breyer. What is driving this result is that Stevens almost always votes for the liberal alternative in every decision. The two probabilistic models compensate for this by pushing Stevens well away from Breyer.

This can be seen in Table 2 which shows the first 40 non-unanimous complete (all 9 justices participating) votes for the 9 justices sorted in the rank order found by OC. We have set the polarities so that “1” is always a vote for the “left” outcome and a “6” is always a vote for the “right” outcome. We have arranged the 40 votes to approximate a Gutman scale-style Simplex display.

Table 2. First 40 Votes of The Rehnquist Court Since Ginsburg

STEVENS	1111111111111111111111111111111111111111..
BREYER	6666666616661111116161111161111166111111..
GINSBURG	6666666161166111111161111111111111111111..
SOUTER	6666661666616611111116111161611111111111..
KENNEDY	6666666666666666611611611116161111111111..
OCONNOR	6666666666666666666116161161611166111611..
REHNQUIST	6666666666666666666611666616611111661111..
SCALIA	6666666666666666661666666611166611611111..
THOMAS	6666666666666666666666666616166611166166..

In the 280 (our data stops at 2001) complete non-unanimous votes, Stevens only votes “6” or conservative 9 times. In contrast Thomas votes “1” or liberal 38 times. On these 280 votes OC correctly classifies 93.1 percent of the 2,520 choices.

What our simple Supreme Court example shows is that there is a non-trivial interaction between the level of error and the interval properties of ideal points. The W-NOMINATE and QN probabilistic methods produce sensible locations for the justices *other than Stevens* but this leaves hanging in the air what to do about the end-point or sag problem, let alone what the relationship is between the level of error and the interval properties in general.

4. A Preliminary Monte-Carlo Analysis of Perfect Voting

With respect to one dimension, the puzzle of how error interacts with the underlying geometry can be summarized in two closely related questions: 1) How high does the error level have to be so that *interval scale* information is recovered; and 2) Under what circumstances will the true rank ordering be recovered? We believe that the answer to this first question crucially depends upon the answer to the second question -- a proof of the *Quinn Conjecture*. The Quinn Conjecture is:

> If the voting space is one dimensional and the noise process is symmetric, then as the number of roll calls goes to infinity the true rank ordering of the legislators will be recovered.

Stated more technically, suppose the error, ε, is drawn from some continuous symmetric probability distribution with mean 0 and finite variance σ^2 having support on the real line; the Quinn Conjecture is:

> If $s = 1$ and $\varepsilon \sim f(0, \sigma^2)$ with $-\infty < \varepsilon < \infty$ and $\sigma^2 < \infty$, then as $q \rightarrow \infty$,
>
> $\mathbf{P(}X_1 < X_2 < X_3 < \ldots < X_{p-2} < X_{p-1} < X_p \mid \mathbf{V)} \rightarrow 1$

Where **V** is the p by q matrix of observed votes and the legislator indices have been permuted so that they correspond to the true ordering of the ideal points.

We performed a very preliminary study of the relationship between the level of error and the interval properties of the corresponding ideal point estimators in one dimension. Table 3 presents the results of one dimensional Optimal Classification output derived from simulated roll call data. The output in the left panel is generated via the QN model, with quadratic utility, and the right panel from the NN model, with normal utility. Each has normally distributed error. The data are designed to produce similar levels of overall error for given number of legislators and roll calls. This error level, the proportion of the total choices incorrectly classified, is reported in the first column of each panel. The second column reports the average majority margin across the set of roll calls. The third column reports the Spearman rank order correlation between the true rank ordering of the legislators and that obtained by the OC algorithm.

For 9, 20, and 50 legislators, Table 3 shows that as the number of roll calls in each model is increased the correlation between the true and reproduced rank order converges to unity, at a roughly similar legislator-to-vote ratio for each piece. The error used in this demonstration is relatively high—ranging from .18 to .29.[3] This allows us to illustrate the impact of the increase in roll calls even given a high degree of noise. For example, at this level of error, 50 legislators voting 98 times produces a rank order that correlates with the true order at .857. Increasing the number of votes to only 245 allows, loosely speaking, a 96.9 percent "recovery" of the true rank order. Near unity between the two rankings is achieved at 4900 votes. Table 3 shows how this relationship behaves under 20 and 9 legislators. In each model, the true rank order steadily emerges from the classification output as the number of legislators and the number of votes diverge. Table 4 provides an example of the true and reproduced rank orderings across three instances of 9 legislators, analogous to those used in bottom panel of Table 3, where shaded cells indicate differences between the two.

Table 3 : Correlation between True and Estimated First Dimension Rankings, varying the number of Roll Calls

		Normal (NN)			Quadratic (QN)		
No. Leg.	No. RCs	Overall Error	Average Majority Margin	True/Estim. Correlation	Overall Error	Average Majority Margin	True/Estim. Correlation
50	98	0.28	0.61	0.857	0.28	0.62	0.915
50	245	0.29	0.61	0.969	0.29	0.62	0.955
50	490	0.28	0.61	0.985	0.29	0.62	0.984
50	1715	0.29	0.61	0.994	0.29	0.62	0.992
50	4900	0.29	0.61	0.998	0.29	0.62	0.998
20	57	0.24	0.62	0.914	0.23	0.62	0.892
20	285	0.26	0.63	0.961	0.25	0.64	0.902
20	475	0.24	0.63	0.982	0.24	0.64	0.971
20	950	0.25	0.62	0.989	0.24	0.63	0.995
20	2850	0.24	0.62	0.999	0.25	0.63	0.999
20	3800	0.25	0.62	1.000	0.24	0.63	0.997
9	80	0.19	0.63	0.817	0.19	0.65	0.867
9	159	0.20	0.64	0.850	0.18	0.65	0.917
9	316	0.18	0.65	0.967	0.18	0.66	0.933
9	789	0.19	0.66	0.983	0.19	0.66	0.967
9	1180	0.19	0.65	1.000	0.19	0.66	1.000

Table 4: Examples of Reproduced and True Rank Orders varying Vote Totals, 9 legislators

158 Votes			793 Votes			1193 Votes		
True Rank Order	Reprod. Rank Order	Prop. correctly classified	True Rank Order	Reprod. Rank Order	Prop. correctly classified	True Rank Order	Reprod. Rank Order	Prop. correctly classified
1	1	0.82	1	1	0.81	1	1	0.79
2	2	0.79	2	3	0.80	2	2	0.79
3	3	0.82	3	2	0.79	3	3	0.81
4	4	0.80	4	4	0.82	4	4	0.83
5	7	0.90	5	5	0.89	5	5	0.88
6	6	0.79	6	6	0.82	6	6	0.81
7	5	0.77	7	7	0.80	7	7	0.81
8	9	0.84	8	8	0.76	8	8	0.78
9	8	0.84	9	9	0.80	9	9	0.81

Table 5 presents a demonstration for 9 legislators similar to that in Table 3, but with an error level (7-11 percent) better approximating that found in most modern legislatures, including the U.S. House, the Senate and, most relevant to our purpose here, the Supreme Court. The change in the correlation between the true and reproduced rank ordering again demonstrates the effect of increasing the number of roll calls. At this level of error, an instance of a perfect correlation is achieved at 160 votes.

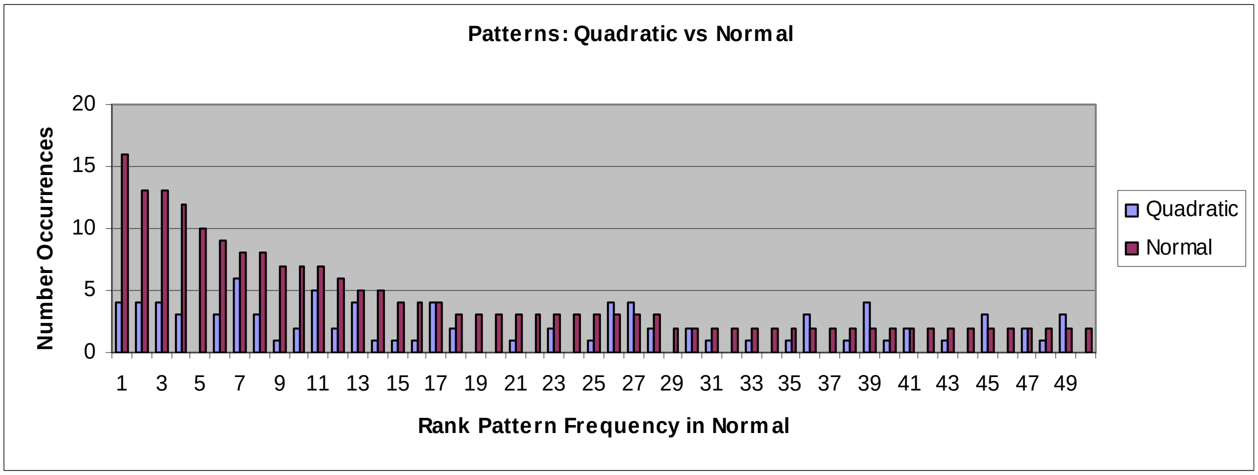
Table 5: Low Error, Normal Model, 9 Individuals

		Normal (NN)			Quadratic (QN)		
No. Leg.	No. RCs	Overall Error	Average Majority Margin	True/Estim. Correlation	Overall Error	Average Majority Margin	True/Estim. Correlation
9	16	0.07	0.66	0.870	.07	.76	.912
9	39	0.08	0.70	0.946	.08	.73	.983
9	78	0.11	0.69	0.967	.09	.70	.983
9	160	0.11	0.67	0.983	.09	.69	1.000
9	307	0.09	0.67	1.000	.10	.66	1.000

5. An Analysis of Voting Patterns: QN, NN and the Supreme Court Compared

The analysis above shows that the effect of the increase in roll calls on the rank-order recovery is consistent across votes generated by either the QN or NN models. However, the patterns generated in the data between the two models are quite different. In this section we analyze all possible patterns generated by nine legislators and compare these with our simulated data and the actual Supreme Court votes we discussed above. Figure 1 presents a comparison between the QN and NN generated roll calls in terms of the frequency of each unique pattern for nine legislators ($2^9 - 2 = 510$, the two unanimous patterns are discarded). Each bar represents the frequency count (out of 280) of each unique pattern generated the QN and NN models. The horizontal axis represents the rank-frequency of the NN model patterns. We have truncated the 510 total unique patterns to the 50 most frequent NN patterns for display purposes. The Figure 1 shows that the differences between the two models arise from the tendency of the NN model to produce "perfect" patterns in much higher frequencies because of its quasi-concave shape (less error in the tails). The pattern frequencies under the QN model are much less concentrated in this range.

Figure 1



In Figures 2 and 3 we compare the actual Supreme Court data used above to simulated data consisting of 280 votes with 7 percent error (the same level of error in the Supreme Court data). In these figures, the horizontal axis is sorted by the rank-frequency of each pattern in the Supreme Court data. Figures 2 and 3 show simulated QN and NN votes, respectively, vis a vis the actual Supreme Court votes. A comparison of Figures 2 and 3 shows that the patterns most frequent in the NN-generated data (the more "perfect" patterns) also occur at a high rate in the Supreme Court data. For these patterns, the frequencies are considerably higher than in the QN data. In sum, the NN model appears to provide a closer approximation of the actual pattern frequencies observed in the recent Supreme Court.

Figure 2

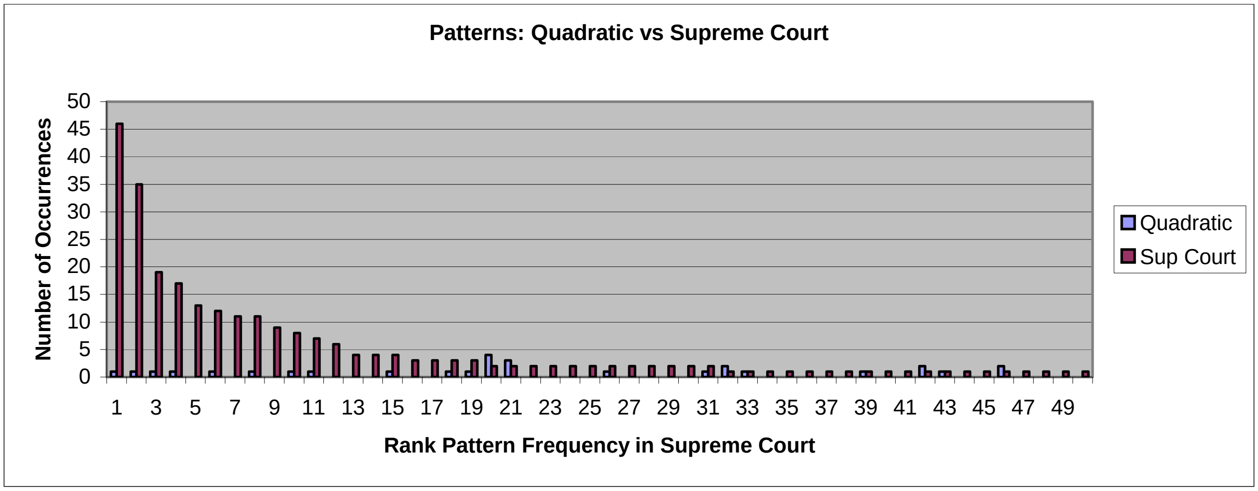
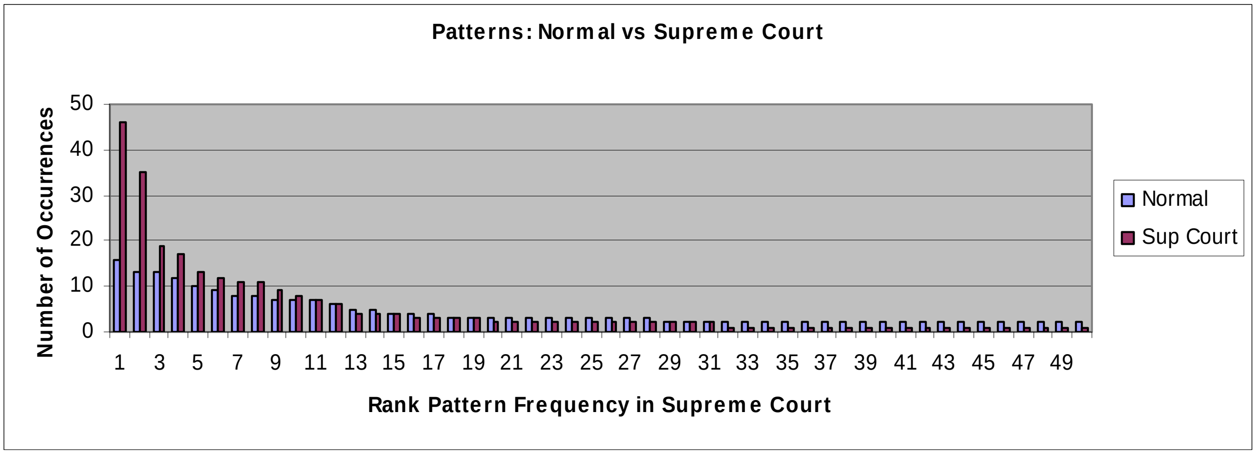


Figure 3



6. Conclusion

In this paper we perform a preliminary exploration of the problems posed by low levels of voting error in the recovery of interval level parameter estimates in spatial (geometric) models of parliamentary voting. Our results, though limited, show strong support for the Quinn Conjecture, namely, if the voting space is one dimensional and the noise process is symmetric, then as the number of roll calls goes to infinity the true rank ordering of the legislators will be recovered.

Although our results for the Quinn Conjecture are consistent across the quadratic and normal utility models, we offer some preliminary findings on the potential differences between the quadratic and normal utility functions.

We also show that the "sag" problem – errorless voting by a legislator at the end of the dimension – is potentially a very serious problem for probabilistic spatial models of small voting bodies such as the U.S. Supreme Court. Specifically, the fact that Justice Clarence Thomas made four times the voting "errors" as Justice Stevens results in Thomas's ideal point being recovered only a short distance away from Justice Scalia while Justice Stevens is nearly half the span of the dimension from Justice Breyer.

We have no magic solution for this problem. Our inclination is to suggest, in a Bayesian spirit, the use of a strong prior distribution based in a substantive argument about the legislators in question to handle the end-points problem. Otherwise, researchers who use geometric models to represent legislatures in one dimension risk making inferences based upon faulty interval properties of their spatial representations.

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Endnotes

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[1] For a survey see Poole (2005) and Poole and Rosenthal (2006).

[2] Within the field of psychometrics the basic result of Poole's theorem is well known. In Guttman scaling a *perfect simplex* is essentially the same as a perfect roll call matrix in one dimension. However, a perfect simplex has a natural polarity – for example, the "Yeas" are always on the same side of the cutting points. Poole's theorem is very similar to Schonemann's (1970) solution for the perfect simplex problem. Schonemann's solution builds upon Guttman's (1954) analysis of the problem. Poole's approach is different in that he analyzed it in the form of agreement scores calculated from roll call votes.

[3] By comparison, the 109th U.S. House, with 608 roll calls and 435 legislators, incorrectly classified only 6.8 percent of votes. However, in the early 19th century, error levels in the House comparable to those used in the table were the norm (Poole and Rosenthal 1997).